

P is a projection onto a line ℓ through the origin.

- (1) If position vector \vec{u} is perpendicular to ℓ , what is $P\vec{u}$? $P\vec{u} = \vec{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- (2) If position vector \vec{v} is on ℓ , what is $P\vec{v}$? $P\vec{v} = \vec{v}$

Find a linear combination of $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$
that equals $\begin{bmatrix} 4 \\ -1 \\ -3 \end{bmatrix}$. Try doing this without
setting up a matrix and "rref"ing!

$$c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ -3 \end{bmatrix}$$

5
2
-3

$$S_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

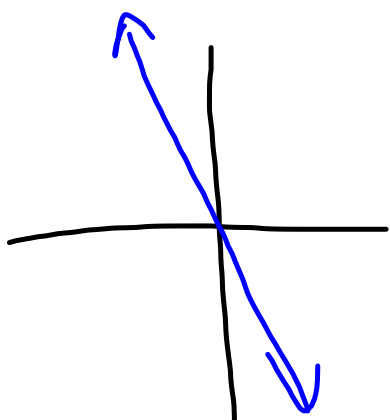
$$\begin{bmatrix} 5 \\ 2 \\ -3 \end{bmatrix}_{S_1} = \begin{bmatrix} 4 \\ -1 \\ -3 \end{bmatrix}_{S_2}$$

$$\begin{aligned} \begin{bmatrix} 4 \\ -1 \\ -3 \end{bmatrix} &= \langle 4, -1, 3 \rangle = 4\vec{i} - \vec{j} - 3\vec{k} \\ S_2 &= \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} = 4 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - 1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - 3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{aligned}$$

$$\left. \begin{array}{l} 2x_1 - 3x_2 + 5x_3 = 7 \\ x_1 + 4x_2 - x_3 = -2 \\ x_1 - 2x_2 + 4x_3 = 1 \end{array} \right\} x_1 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 4 \\ -2 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \overset{A}{\begin{bmatrix} -3 & 5 \\ 4 & -1 \\ -2 & 4 \end{bmatrix}} \overset{\vec{x}}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \overset{\vec{b}}{\begin{bmatrix} 7 \\ -2 \\ 1 \end{bmatrix}}$$

$$A\vec{x} = \vec{b}$$



$$P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$5a - 3b = 2, a - 7b = 2$$

$$\frac{9}{17}$$