Discuss the solution to $\boxed{A\vec{x} = \vec{0}.}$
(no solution, one solution, inf many sols?)
What if $\underline{\underline{\det(A) = 0}}$? $\underline{\underline{\det(A) \neq 0}}$?

$A\vec{x} = \vec{b}$

$\therefore \vec{x} = A^{-1}\vec{b}$

A is not invertible

A is invertible

$A = \begin{bmatrix} 3 & -1 \\ -4 & 0 \end{bmatrix}$, $\vec{u} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

Find $A\vec{u}$, $A\vec{v}$, $A\vec{w}$. What do you notice?

$A\vec{u} = \begin{bmatrix} -4 \\ 4 \end{bmatrix}$, $A\vec{v} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, $A\vec{w} = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$

$$A\vec{u} = 4\vec{u}$$

$$A\vec{w} = -\vec{w}$$

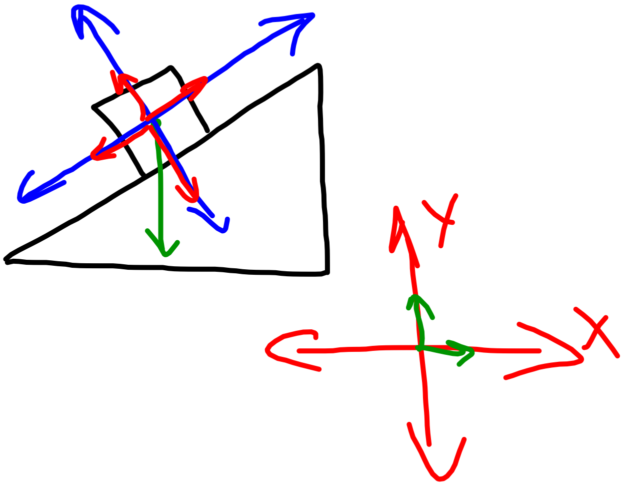
$$\begin{bmatrix} & \end{bmatrix}\begin{bmatrix} \\ \end{bmatrix} = \alpha \begin{bmatrix} \\ \end{bmatrix}$$

We say "$\vec{u}$ is an eigenvector of A with eigenvalue $\lambda = 4$."

w is an eigenvector of A with eigenvalue $\lambda = -1$.

$$A = \begin{bmatrix} 3 & -1 \\ -4 & 0 \end{bmatrix}, \vec{u} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \vec{w} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$A \begin{bmatrix} -2 \\ -8 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$$



A <u>nonzero</u> vector $\vec{x}$ is an eigenvector of A with eigenvalue λ if $A\vec{x} = \lambda\vec{x}$.

$$A = \begin{bmatrix} 3 & -1 \\ -4 & 0 \end{bmatrix}$$

How do we find eigenvalues and eigenvectors?

This only has a solution if $\det(A-\lambda I)=0$

$$A\vec{x} = \lambda\vec{x}$$

$$A\vec{x} - \lambda\vec{x} = \vec{0}$$

$$A\vec{x} - \lambda I\vec{x} = \vec{0}$$

$$\underbrace{(A - \lambda I)}\vec{x} = 0$$

$$B\vec{x} = \vec{0}$$

To find eigenvalues of A, we solve $\det(A-\lambda I)=0$

$$A-\lambda I = \begin{bmatrix} 3 & -1 \\ -4 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 3-\lambda & -1 \\ -4 & 0-\lambda \end{bmatrix}$$

$$\det(A-\lambda I) = (3-\lambda)(0-\lambda) - (-4)(-1)$$

$$= -3\lambda + \lambda^2 - 4$$

$$= \lambda^2 - 3\lambda - 4$$

$\det(A - \lambda I) = 0$

$\lambda^2 - 3\lambda - 4 = 0$

$(\lambda+1)(\lambda-4) = 0$

$\boxed{\lambda = -1, 4}$

eigenvalues

Eigenvector for $\lambda = -1$:

Solve $(A - \lambda I)\vec{x} = \vec{0}$ for $\lambda = -1$.

$$A - \lambda I = \begin{bmatrix} 3-(-1) & -1 \\ -4 & -(-1) \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -1 \\ -4 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -1 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$4x_1 - x_2 = 0$

$x_1 = 1, x_2 = 4$

$\vec{x} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ is an eigenvector of A with eigenvale -1.

$$\begin{bmatrix} 2 & 5 \\ -2 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 & 0 \\ -2 & -5 & 0 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & \frac{5}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$2x_1 + 5x_2 = 0$

$x_1 = 5 \quad x_2 = -2$

x_2 is free

Let it be 2

$x_1 + \frac{5}{\cancel{2}}(\cancel{2}) = 0$

$x_1 = -5$