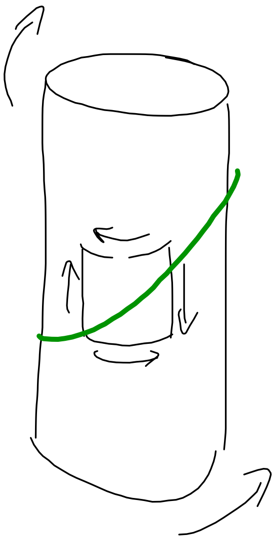
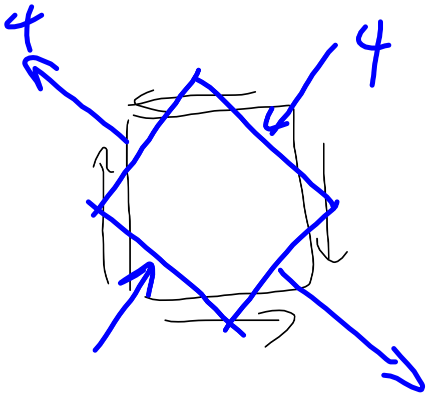




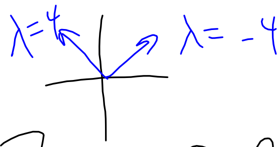
$$\begin{bmatrix} 0 & -4 \\ -4 & 0 \end{bmatrix} = A$$

$$A-\lambda I=\begin{bmatrix} 0 & -4 \\ -4 & 0 \end{bmatrix}-\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}=\begin{bmatrix} -\lambda & -4 \\ -4 & -\lambda \end{bmatrix}$$

$$\det(A-\lambda I)=\lambda^2-16=(\lambda+4)(\lambda-4)=0$$

$$\lambda=-4,4$$ principal stresses

$$(A - \lambda I)\vec{x} = \vec{0}$$



for $\lambda = -4$: $\begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Longrightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

for $\lambda = 4$: $\begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Longrightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & -4 \\ -4 & 0 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} -4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$A = PDP^{-1} \iff D = P^{-1}AP$$

$$P = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{\sqrt{2}}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \|\vec{v}\| = \sqrt{1^2 + 1^2} = \sqrt{2} \qquad \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

Give a function $y=y(x)$ whose derivative is -3 times the function itself. Is there more than one such function?

$$\cancel{y = x^2}$$
$$\cancel{y' = 2x}$$

$$y = e^{-3x}$$
$$y' = -3e^{-3x}$$

$$y = -\frac{3x^2}{2}$$
$$y' = -\frac{6x}{2}$$
$$y' = -3x$$

$y = 5e^{-3x}$

$y' = -15e^{-3x}$

$y' = -3(5e^{-3x})$

$y = Ce^{-3x}$

$\boxed{y' = -3y}$ Differential equation

Solution is

$y = Ce^{-3x}$

$5x + 7 = 7$

$\boxed{y' = -3y \quad y(0) = 2}$ says $y = 2$ when $x = 0$

IVP

$y = Ce^{-3x}$

$2 = Ce^{-3(0)}$

$2 = C$

$y = 2e^{-3x}$

$x' = -3x, \quad x(0) = 2 \quad$ time (t)

$$\boxed{\begin{aligned} x_1' &= x_1 + 2x_2 & x_1(0) &= 10 \\ x_2' &= 3x_1 + 2x_2 & x_2(0) &= 5 \end{aligned}}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \Longrightarrow P = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}, D = \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A = PDP^{-1}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$\vec{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = P^{-1}\vec{x}$

$x_1 = x_1(t)$

$x_2 = x_2(t)$

$\vec{y}' = D\vec{y}$

$\vec{x} = P\vec{y}$

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$\vec{x}' = A\vec{x}$

$\vec{x}' = PDP^{-1}\vec{x}$

$P^{-1}\vec{x}' = DP^{-1}\vec{x}$

$y_1' = 4y_1$

$y_2' = -y_2$

$y_1 = C_1 e^{4t}$

$y_2 = C_2 e^{-t}$

$C_1 = 3$

$C_2 = -4$

$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}, \quad \vec{y} = P^{-1}\vec{x}$

$\vec{y}(0) = P^{-1}\vec{x}(0)$

$= \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

$$\vec{y} = \begin{bmatrix} 3e^{4t} \\ -4e^{-t} \end{bmatrix} \quad \vec{x} = P\vec{y} = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 3e^{4t} \\ -4e^{-t} \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6e^{4t} + 4e^{-t} \\ 9e^{4t} - 4e^{-t} \end{bmatrix}$$