① $f(x) = x^2 - 2x + 5$. Find $f(3)$.

② $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ 2x_1 \\ x_1 - x_2 \end{bmatrix}$. Find $T\left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

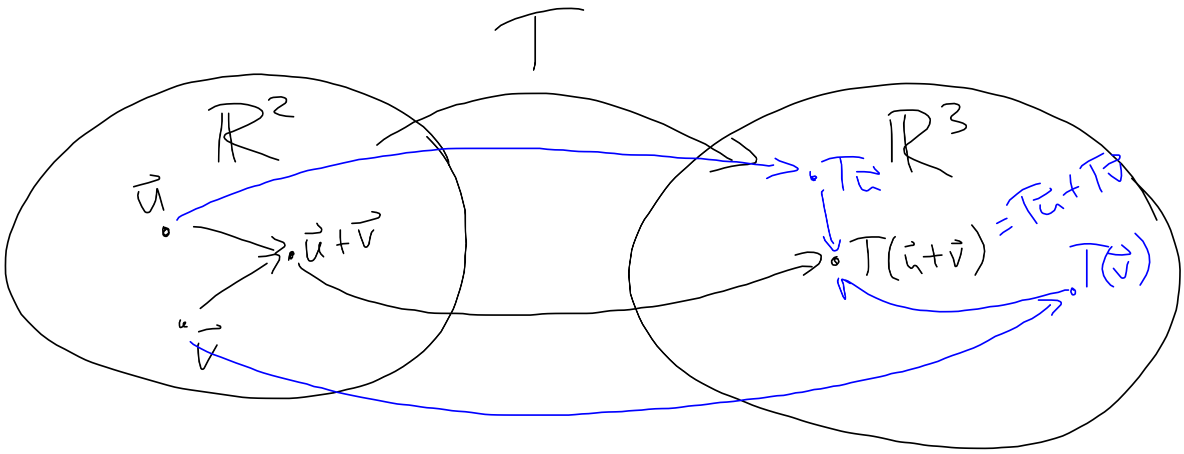
Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$f(x) = x^2 - 2x + 5$.

Define $T: \mathbb{R}^2 \to \mathbb{R}^3$ by

(3) For the f from #1, find $f(1)+f(3)$ and $f(1+3)$. Are they equal?

(4) For the T from #2, find $T\left(\begin{bmatrix}1\\2\end{bmatrix}\right)+T\left(\begin{bmatrix}3\\0\end{bmatrix}\right)$ and $T\left(\begin{bmatrix}1\\2\end{bmatrix}+\begin{bmatrix}3\\0\end{bmatrix}\right)$. Are they equal?


T
$\mathbb{R}^2$
$\vec{u}$
$\vec{v}$
$\vec{u}+\vec{v}$
$\mathbb{R}^3$
$T\vec{u}$
$T(\vec{u}+\vec{v})$
$= T\vec{u} + T\vec{v}$
$T(\vec{v})$

$f(a+b) = f(a) + f(b)$

not usually true

$(x+3)^2 = x^2 + 9$

$\sqrt{x^2+y^2} = \sqrt{x^2} + \sqrt{y^2} = x + y$

$f(cx) = cf(x)$

$\sin(a+b) = \sin a + \sin b$

$\sin(a+b) =$

$f(x) = 3x$

$$\frac{d}{dx}(x^2+5x-2) = 2x+5$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(5x) + \frac{d}{dx}(-2)$$

$$+ 5\frac{d}{dx}(x)$$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ 2x_1 \\ x_1 - x_2 \end{bmatrix}$$

Is $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$?

$$T(\vec{u}+\vec{v}) = T\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right) = T\left(\begin{bmatrix} u_1+v_1 \\ u_2+v_2 \end{bmatrix}\right) =$$

$$\begin{bmatrix} (u_1+v_1) + (u_2+v_2) \\ 2(u_1+v_1) \\ (u_1+v_1) - (u_2+v_2) \end{bmatrix} =$$

$$T(\vec{u}) + T(\vec{v}) = T\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + T\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} u_1+u_2 \\ 2u_1 \\ u_1-u_2 \end{bmatrix} + \begin{bmatrix} v_1+v_2 \\ 2v_1 \\ v_1-v_2 \end{bmatrix} =$$

$$= \begin{bmatrix} (u_1+u_2)+(v_1+v_2) \\ 2u_1+2v_1 \\ (u_1-u_2)+(v_1-v_2) \end{bmatrix}$$

$S: \mathbb{R}^3 \to \mathbb{R}^2$ is defined by $S\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 x_2 \\ x_2 x_3 \end{bmatrix}$

$S(\vec{u}+\vec{v}) \stackrel{?}{=} S\vec{u} + S\vec{v}$ Let $\vec{u} = \begin{bmatrix} \\ \\ \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} \\ \\ \end{bmatrix}$

$S(\vec{u}+\vec{v}) = \cdots = \begin{bmatrix} \\ \end{bmatrix}$ and $S\vec{u} + S\vec{v} = \cdots = \begin{bmatrix} \\ \end{bmatrix}$

$S(\vec{u}+\vec{v}) \neq S\vec{u} + S\vec{v}$, S is not a linear transformation.

Let $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, define $T_A : \mathbb{R}^2 \to \mathbb{R}^2$

by $T_A\vec{x} = A\vec{x}$. $T(c\vec{x}) = cT\vec{x}$

$c=5$, $\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $T_A(c\vec{x})$ $cT_A\vec{x}$

$$T_A(c\vec{x}) = T_A\left(5\begin{bmatrix}2\\3\end{bmatrix}\right) = A\begin{bmatrix}10\\15\end{bmatrix} = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}10\\15\end{bmatrix} = \begin{bmatrix}40\\90\end{bmatrix}$$

$$cT_A\vec{x} = 5\,T_A\begin{bmatrix}2\\3\end{bmatrix} = 5\left(\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix}\right) = 5\begin{bmatrix}8\\18\end{bmatrix} = \begin{bmatrix}40\\90\end{bmatrix}$$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ 2x_1 \\ x_1 - x_2 \end{bmatrix}$$

$$[T] = \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ 2x_1 \\ x_1 - x_2 \end{bmatrix}$$

T is the transformation

$[T]$ is the matrix that does the transformation