

① The line in \mathbb{R}^3 through the origin and $(-2, 1, 5)$ is a subspace of \mathbb{R}^3 .
Give a basis for it.

$$\left\{ t \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix} \mid t \in \mathbb{R} \right\} \quad \mathcal{B} = \left\{ \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix} \right\}$$

② The plane in \mathbb{R}^3 containing the z -axis and $(-2, 1, 5)$ is a 2D space of \mathbb{R}^3 .
Give a basis for it.

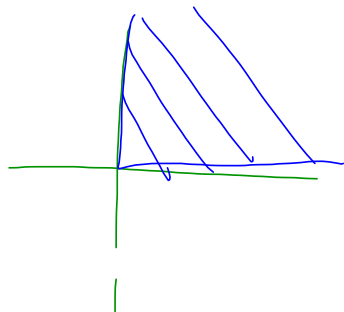
$$B = \left\{ \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$



$$s \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A\vec{x} = \vec{0} \quad \vec{x} = \vec{0} \text{ is always a sol.}$$

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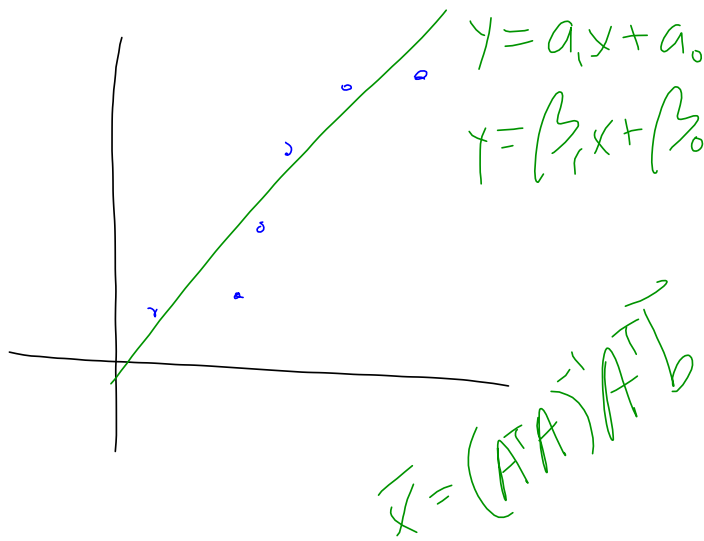


$$\vec{x} = s \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} + t \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \quad \mathcal{B} = \left\{ \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}, \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \right\}$$

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3x_1 - x_2 \\ x_1 + x_2 \end{bmatrix}, \quad [T] = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$$

$$T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [T] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



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$$\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} \vec{b} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \\ 4 & 8 \end{pmatrix}$$

$$S \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ 2x \\ -3y \end{bmatrix} \quad T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5x-y \\ x+4y \end{bmatrix}$$

$S \circ T$ ~~$T \circ S$~~ $S \circ T \begin{bmatrix} x \\ y \end{bmatrix} = S \left(T \begin{bmatrix} x \\ y \end{bmatrix} \right)$

$$= S \left(\begin{bmatrix} 5x-y \\ x+4y \end{bmatrix} \right)$$

$$= \begin{bmatrix} (5x-y) + (x+4y) \\ 2(5x-y) \\ -3(x+4y) \end{bmatrix}$$

$$= \begin{bmatrix} 6x+3y \\ 10x-2y \\ -3x-12y \end{bmatrix}$$

$$[S \circ T] = [S][T]$$