Math 341 EXAM 1, WINTER 2017 **Name:** ____________________

Unless stated otherwise, each numbered exercise is worth six points.

1. Consider the system $\begin{aligned} x - 2y - 3z &= -1 \\ 2x + y + z &= 6 \\ x + 3y - 2z &= 13 \end{aligned}$.

   (a) **Using your calculator**, the solution to the system

   is $x =$ ______ , $y =$ ______ , $z =$ ______

   (b) In the space to the right, give the linear combination form of the system.

2. (a) Give the row operation used to obtain the zero in the second row, first column, and fill in the blanks for the resulting matrix.

$$\begin{bmatrix} 1 & 5 & -3 & 7 \\ -3 & 0 & 3 & -4 \\ 2 & 6 & 1 & 2 \end{bmatrix} \quad \xRightarrow{\hspace{4em}} \quad \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ 0 & \_\_ & \_\_ & \_\_ \\ 2 & 6 & 1 & 2 \end{bmatrix}$$

   (b) Give the row operation used to obtain the zero in the bottom row, second column, and fill in the blanks for the resulting matrix. **This is not from the same system of equations as the matrices in part (a).**

$$\begin{bmatrix} 1 & -1 & 4 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 1 & 4 \end{bmatrix} \quad \xRightarrow{\hspace{4em}} \quad \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ 0 & \_\_ & \_\_ & \_\_ \\ 0 & 0 & \_\_ & \_\_ \end{bmatrix}$$

3. Circle the letters of all of the following that are linear equations. *3 points*

   (a) $y = ax^3 + bx^2 + cx + d$ for $x = -2$

   (b) $y = ax^3 + bx^2 + cx + d$ for $a = 2$, $b = -1$, $c = -5$, $d = 4$

   (c) $t_3 = \dfrac{t_1 + t_4 + 48}{4}$

4. There is a unique (meaning only one) polynomial

$$y = a + bx + cx^2 + dx^3 + \cdots$$

   of some degree whose graph goes through the points

$$(1, 2)\ , \qquad (4, 5)\ , \qquad (6, -3).$$

   (a) The degree of the polynomial is (circle one)

   1 2 3 4 5 *1 point*

   (b) In the space to the right, give the system used to find the coefficients of the polynomial - **don't solve it!** *5 points*

5. Each of the following matrices is the row-reduced matrix resulting from the augmented matrix for a system of equations. For each, assume the unknowns are x_1, x_2, etc. and

- if there is a single solution, give it **just to the right of the matrix** *1 point*
- if there is no solution, write NS to the **left** of the letter *1 point*
- if there are infinitely many solutions, give the general solution and **TWO** particular solutions in the remaining space to the right. *4 points*

(a) $\begin{bmatrix} 1 & 3 & 0 & 0 & -7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 3 & 0 & -2 & -7 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 0 & -7 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

6. Three equations of relations in $\mathbb{R}^3$ are given below. For each, determine first whether the equation is that of a plane or not. If it is, circle the word *plane*; if it is not, cross out the word plane. Then, **for those that ARE planes**, give the points at which they intersect each axis; if an axis is not intersected, leave the blank empty.

(a) $2x - y + 3z = 6$

plane

x: ______________

y: ______________

z: ______________

(b) $x^2 - 3y + z = 4$

plane

x: ______________

y: ______________

z: ______________

(c) $x - 2z = 4$

plane

x: ______________

y: ______________

z: ______________

7. There is only one linear combination of the three vectors $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{u}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ that equals the vector $\mathbf{w} = \begin{bmatrix} -2.4 \\ 10.0 \\ 2.2 \end{bmatrix}$. Find and give it - **be sure to write your answer as a linear combination!**

8. Give the vector *equation* of the plane in $\mathbb{R}^3$ containing $P(1,-3,4)$, $Q(0,5,2)$ and $R(-7,10,4)$.

9. The general solution to a system of equations is

$$\begin{aligned} x_1 &= -3+4r+2s-5t \\ x_2 &= r \\ x_3 &= 2-s+4t \\ x_4 &= -s+3t \\ x_5 &= s \\ x_6 &= t \end{aligned}$$

   (a) In the space to the right, give the vector form of the solution. *4 points*

   (b) The set of all particular solutions represents a ______ dimensional "plane" (which we call a *hyperplane*) in ______ dimensional space. *2 points*

10. (a) The set $\mathcal{S} = \left\{ \begin{bmatrix} a \\ b \\ ab \end{bmatrix} \in \mathbb{R}^3 \right\}$

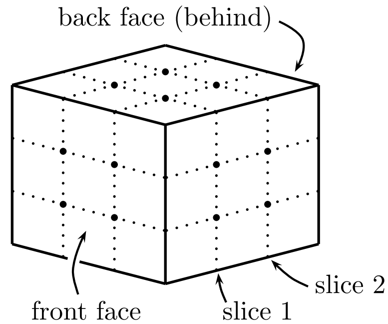
- (circle one) is is not closed under addition and
- is is not closed under scalar multiplication.

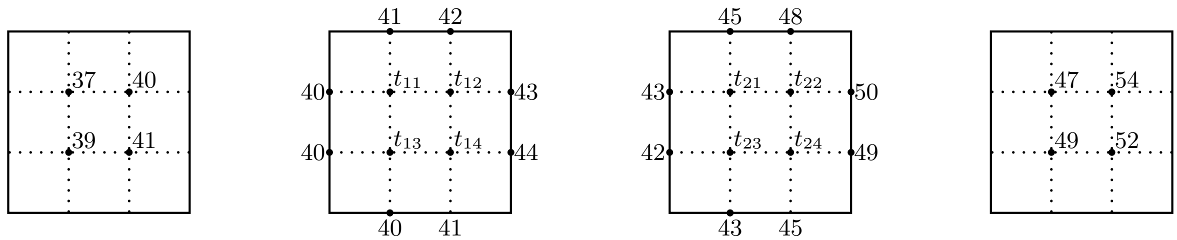
   (b) The set $\mathcal{S} = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \;\middle|\; y \geq 0 \right\}$

- is is not closed under addition and
- is is not closed under scalar multiplication.

Do **EXACTLY ONE** of the following **in the space at the bottom of the page**. Circle the number of the one that you want me to grade.

11. A cube of some solid material is shown to the right, and the grids below show temperatures, known or unknown, at all nodes on the front face, each of the two slices, and the back face. Give an equation for the equilibrium temperature t_{24}, in terms of other (known) boundary temperatures and (unknown) interior equilibrium temperatures. **Assume that the faces and slices shown below are all oriented the same way, so the points with temperatures** $41°$**,** t_{14}**,** t_{24} **and** $52°$ **are all on the same line.** In three dimensions, the equilibrium temperature at an interior point is the average of all the temperatures on a *sphere* centered at the point.





front face | slice 1 | slice 2 | back face

12. There are infinitely many linear combination of $\mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 4 \\ 0 \\ 5 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} -2 \\ -7 \\ 1 \end{bmatrix}$ that equal the vector $\mathbf{w} = \begin{bmatrix} -4 \\ -12 \\ 1 \end{bmatrix}$. Find *ONE* and give it - **be sure to write your answer as a linear combination!**

13. Consider the vector equation $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + s\mathbf{u} + t\mathbf{v}$, with $s, t \in \mathbb{R}$.

    (a) Give vectors $\mathbf{u}$ and $\mathbf{v}$ for which the set of all such vectors $\mathbf{x}$ is a plane through the origin.

    (b) Give vectors $\mathbf{u}$ and $\mathbf{v}$ for which the set of all such vectors $\mathbf{x}$ is a line *NOT* through the origin.