**Math 341** EXAM 1, WINTER 2017 **Name:** ________________________________

**Unless stated otherwise, each numbered exercise is worth six points.**

1. Consider the system $\begin{aligned} x - 2y - 3z &= -1 \\ 2x + y + z &= 6 \\ x + 3y - 2z &= 13 \end{aligned}$. Give the linear combination form and matrix form (*NOT* augmented matrix form) of the system in the spaces below.

Linear Combination Form                                                  Matrix Form

2. Find the determinant of the matrix below. **Combine like terms in your answer and put a box around your final answer.**

$$\begin{bmatrix} a & b & c \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

3. (a) For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$, $AA^T =$

   (b) Give two *words* for what kind of matrix $AA^T$ is:

4. Matrix $A$ is $4 \times 3$.

   (a) Give the dimensions of a matrix $B$ for which $AB$ would exist but $BA$ would not: _______ $\times$ _______

   (b) For your answer to part (a), the dimensions of $AB$ would be _______ $\times$ _______

   (c) Give the dimensions of a matrix $C$ for which $AC$ and $CA$ would both exist: _______ $\times$ _______

5. Give a matrix $A$ for which $A\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3x_1 - x_2 \\ 5x_1 + 4x_2 \\ -2x_1 + 3x_2 \end{bmatrix}$:    $A =$

6. Let $A = \begin{bmatrix} -9 & 4 \\ 7 & -3 \end{bmatrix}$. Use $A$ to demonstrate the steps used to find the inverse of a matrix of any size. Indicate clearly what you are doing, and conclude by giving the inverse matrix $A^{-1}$. Use your calculator to do *rref*, but indicate where you are doing that by writing $\stackrel{rref}{\Longrightarrow}$.

7. Determine whether $A = \begin{bmatrix} -3 & 2 \\ -5 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}$ are inverses of each other, *without finding the inverse of either.* Show clearly the computation used to do this in the space below.

$A$ and $B$ (circle one) are are not inverses of each other.

8. Give **all** steps for how an inverse matrix is used to solve a matrix equation $A\mathbf{x} = \mathbf{b}$. Start with the given equation and end with $\mathbf{x} =$ .

9. Recall that $\det(A) = 0$ if, and only if, $A$ is *NOT* invertible. For each of the following scenarios, indicate *ALL possible* number of solutions to $A\mathbf{x} = \mathbf{b}$ by circling all correct choices for each.

(a) $\mathbf{b} = 0$ (nothing known about $\det(A)$) none one infinitely many

(b) $\det(A) = 0$ none one infinitely many

(c) $\det(A) = 0$ and $\mathbf{b} = 0$ none one infinitely many

(d) $\det(A) \neq 0$ and $\mathbf{b} = 0$ none one infinitely many

10. Suppose that, in $\mathbb{R}^2$, $R$ is a rotation by any given angle that is not a multiple of $2\pi = 360°$, $P$ is a projection onto any given line, and $C$ is a reflection across any given line. **Use the space below to draw any pictures that might help you with this exercise and the next one.**

(a) Give all of the matrices whose squares equal $I$:

(b) Give all of the matrices that equal their own squares:

(c) Give **all** the matrices $M$ for which there is at least one nonzero vector $\mathbf{x}$ such that $M\mathbf{x} = \mathbf{x}$:

(d) Give all of the matrices that have inverses:

11. Suppose that $A$ is a $2 \times 2$ matrix. Finish each sentence by circling all transformations that would make the statement true. Assume that rotation means by any angle that is not a multiple of $2\pi = 360°$.

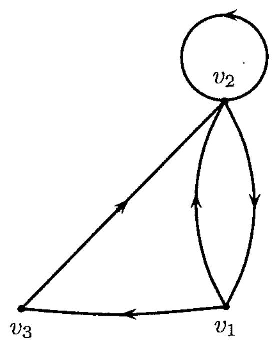
(a) $A^3 = A^5$ if $A$ is a rotation projection reflection

(b) $A$ changes the directions of all vectors if $A$ is a rotation projection reflection

(c) $A = A^{-1}$ if $A$ is a rotation projection reflection

(d) There are *nonzero* vectors $\mathbf{x}$ for which $A\mathbf{x} = \mathbf{0}$ if $A$ is a rotation projection reflection

12. Consider the directed graph shown below and to the left. There are ______ 4-paths from $v_2$ to $v_4$; indicate how you determined this in the space below.




13. For the directed graph in the previous exercise, give two 3-paths (there are more, but just give any two) from $v_1$ to $v_2$, using the standard notation for a path.

14. Let $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and suppose that $A$ is a $2 \times 2$ matrix for which $A\mathbf{e}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and $A\mathbf{e}_2 = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$.

Find the entries of $A$ using this information, **showing clearly how those facts are used.**