Math 341 EXAM 3, WINTER 2017 **Name:** ______________________________

Do **ALL BUT THREE** of the **numbered** exercises. **Cross out the three that you don't want me to grade.** Each exercise is worth six points.

1. Consider the set $S = \left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} \right\}$. In the space provided below, give each of the following.

   (a) A nonzero vector $\mathbf{u}_1$ in the span of S that is not a scalar multiple of either of those vectors.

   (b) Another vector $\mathbf{u}_2$ that *IS NOT* in the span of S.

$\mathbf{u}_1 =$ $\mathbf{u}_2 =$ $\mathbf{u}_3 =$ $\mathbf{u}_4 =$

2. In the space provided above, give each of the following. **You may re-use some of the vectors from the previous exercise.**

   (a) A vector $\mathbf{u}_3$ that could be added to the set to make it a basis for $\mathbb{R}^3$.

   (b) A nonzero vector $\mathbf{u}_4$ that could be added to the set without making it a basis for $\mathbb{R}^3$.

3. Three sets of vectors are given below.

   (a) Circle the letters (a, b , c) of the sets that are **dependent**.

   (b) For the sets that *ARE* dependent, give one of the vectors as a linear combination of the others in the space below. You don't need to write out the vectors - you can refer to them by their names, but use arrows!

(a) $\mathbf{u}_1 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 3 \\ -9 \end{bmatrix}$

(b) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(c) $\mathbf{w}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{w}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{w}_3 = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$

4. Let $S \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xy \\ x+y \end{bmatrix}$ and $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xy \\ x-y \\ 2y \end{bmatrix}$.

   One of $S \circ T$ and $T \circ S$ exists - give it in the space to the right, **using correct notation**.

The matrix $A = \begin{bmatrix} 1 & -2 & 2 & -5 \\ 2 & -4 & 1 & -1 \\ 3 & -6 & 3 & -6 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Use this for the following.

5. Give a nonzero vector $\mathbf{u}$ in the column space of A. (There are *MANY* possibilities - simple is good, but not required!)

$\mathbf{u} =$

6. Determine whether $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 1 \end{bmatrix}$ is in the null space of A, **indicating clearly how you make your determination**. Answer (circle just *is* or *is not*): $\mathbf{w}$ is is not in the null space of A.

7. Give a basis for the column space of A.

$\mathcal{B} =$

8. Give a basis for the null space of A.

$\mathcal{B} =$

8. Consider the transformations $S\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xy \\ x^2 \end{bmatrix}$ and $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x - y \\ x + 2y \\ 5x + 2y \end{bmatrix}$.

   One of the transformations is linear. Give the matrix of the transformation in the space to the right. **Label it, using correct notation.**

9. For the transformation from the previous exercise that is not linear, demonstrate that it is not with a *specific* counterexample. (There are two conditions for a transformation to be linear, and this will violate both of them. Choose one or the other and demonstrate **correctly**.)

10. In the blank next to each set $\mathcal{S}$, tell whether the span of the set is

    (a) a point (b) a line (c) a plane (d) all of $\mathbb{R}^3$

$\mathcal{S}_1 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$ ______ $\mathcal{S}_2 = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \end{bmatrix} \right\}$ ______ $\mathcal{S}_3 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ ______

$\mathcal{S}_4 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ -4 \\ 0 \end{bmatrix} \right\}$ ______ $\mathcal{S}_5 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ ______ $\mathcal{S}_6 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ ______

11. Refer to the sets from the previous exercise. Give a vector meeting each of the following conditions in the appropriate space below. Write *DNE* if no such vector exists.

    (a) A vector in the span of $\mathcal{S}_4$ that is not a scalar multiple of any of the vectors in it.

    (b) A vector that could be added to $\mathcal{S}_6$ to make it a basis for $\mathbb{R}^3$.

    (c) A vector not in the span of $\mathcal{S}_3$.

    (a) (b) (c)

12. Again referring to the sets from the previous two exercises, list all that are linearly independent: ____________

13. For each of the following,

- circle the letters of the ones that are not subspaces
- for any one that is not a subspace, give one of the following reasons why it isn't: (i) it doesn't contain the zero vector, (ii) it is not closed under scalar multiplication, (iii) it is not closed under vector addition

(a) The set of vectors in $\mathbb{R}^3$ of the form $\begin{bmatrix} 2a \\ b \\ a \end{bmatrix}$.

(b) The set of vectors in $\mathbb{R}^3$ of the form $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + t \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}$.

(c) The set of vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ in $\mathbb{R}^2$ for which $xy \leq 0$.

(d) The xz-plane in $\mathbb{R}^3$.

14. Suppose that we are using the least-squares method to find the equation of the line that best approximates the points $(0,1)$, $(1,2)$, $(2,2)$, $(3,4)$. Give the matrix A and the vector $\mathbf{b}$ (label each, of course!) that would be used in the formula $\bar{x} = (A^T A)^{-1} A^T \mathbf{b}$.

15. (a) In the space to the right, give the transformation T (not a matrix!) that translates every point in $\mathbb{R}^2$ three points to the right and one unit down.

(a) Why can the transformation T not be represented by a 2×2 matrix?

(a) In the space to the right, give the matrix that can be used to perform T when the vectors in $\mathbb{R}^2$ are put into homogeneous coordinates.