

Do ALL BUT THREE of the numbered exercises. Cross out the three that you don't want me to grade. Each exercise is worth six points.

1. Consider the set $S = \left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} \right\}$. In the space provided below, give each of the following.

13
6
18

- (a) A nonzero vector \mathbf{u}_1 in the span of S that is not a scalar multiple of either of those vectors.
- (b) Another vector \mathbf{u}_2 that IS NOT in the span of S .

$\mathbf{u}_1 = \begin{bmatrix} a \\ 0 \\ b \end{bmatrix}$ *not a multiple of* $\mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\mathbf{u}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\mathbf{u}_4 = \text{see answers}$ $\begin{bmatrix} a \\ 0 \\ b \end{bmatrix}$

2. In the space provided above, give each of the following. You may re-use some of the vectors from the previous exercise.

- (a) A vector \mathbf{u}_3 that could be added to the set to make it a basis for \mathbb{R}^3 .
- (b) A nonzero vector \mathbf{u}_4 that could be added to the set without making it a basis for \mathbb{R}^3 .

3. Three sets of vectors are given below.

- (a) Circle the letters (a, b, c) of the sets that are **dependent**.
- (b) For the sets that ARE dependent, give one of the vectors as a linear combination of the others in the space below. You don't need to write out the vectors - you can refer to them by their names, but use arrows!

(a) $\mathbf{u}_1 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 3 \\ -9 \end{bmatrix}$

(b) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(c) $\mathbf{w}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{w}_3 = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$

$\vec{u}_2 = -3\vec{u}_1$

$c_1 = -3t$
 $c_2 = 2t$
 $c_3 = t$

$-3t\vec{w}_1 + 2t\vec{w}_2 + t\vec{w}_3 = \vec{0}$

Go from there

$\vec{w}_3 = 3\vec{w}_1 - 2\vec{w}_2$

4. Let $S \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xy \\ x+y \end{bmatrix}$ and $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xy \\ x-y \\ 2y \end{bmatrix}$.

$T \circ S \begin{bmatrix} x \\ y \end{bmatrix} = T \left(\begin{bmatrix} xy \\ x+y \end{bmatrix} \right) = \begin{bmatrix} x^2y + xy^2 \\ xy - x - y \\ 2x + 2y \end{bmatrix}$

One of $S \circ T$ and $T \circ S$ exists - give it in the space to the right, using correct notation.

The matrix $A = \begin{bmatrix} 1 & -2 & 2 & -5 \\ 2 & -4 & 1 & -1 \\ 3 & -6 & 3 & -6 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Use this for the following.

5. Give a nonzero vector \mathbf{u} in the column space of A . (There are *MANY* possibilities - simple is good, but not required!)

$\mathbf{u} =$ See answer

6. Determine whether $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 1 \end{bmatrix}$ is in the null space of A , **indicating clearly how you make your**

determination. Answer (circle just *is* or *is not*): \mathbf{w} is is not in the null space of A .

$$A\vec{w} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

7. Give a basis for the column space of A .

$$B = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \right\}$$

8. Give a basis for the null space of A .

$$B = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \\ 1 \end{bmatrix} \right\}$$

$$\begin{aligned} x_1 &= 2s - t \\ x_2 &= s \\ x_3 &= 3t \\ x_4 &= t \end{aligned}$$

$$\vec{x} = s \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

8. Consider the transformations $S \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xy \\ x^2 \end{bmatrix}$ and $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x - y \\ x + 2y \\ 5x + 2y \end{bmatrix}$. $[T] = \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 5 & 2 \end{bmatrix}$

One of the transformations is linear. Give the matrix of the transformation in the space to the right. **Label it, using correct notation.**

9. For the transformation from the previous exercise that is not linear, demonstrate that it is not with a *specific* counterexample. (There are two conditions for a transformation to be linear, and this will violate both of them. Choose one or the other and demonstrate **correctly**.)

See example

10. In the blank next to each set S , tell whether the span of the set is

(a) a point

(b) a line

(c) a plane

(d) all of \mathbb{R}^3

$S_1 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$ b $S_2 = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \end{bmatrix} \right\}$ b $S_3 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ d

$S_4 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ -4 \\ 0 \end{bmatrix} \right\}$ c $S_5 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ a $S_6 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ c

11. Refer to the sets from the previous exercise. Give a vector meeting each of the following conditions in the appropriate space below. Write *DNE* if no such vector exists.

(a) A vector in the span of S_4 that is not a scalar multiple of any of the vectors in it.

(b) A vector that could be added to S_6 to make it a basis for \mathbb{R}^3 .

(c) A vector not in the span of S_3 .

(a)

(b)

(c)

See answers

12. Again referring to the sets from the previous two exercises, list all that are linearly independent: S_1, S_3, S_5, S_6

Maybe not

13. For each of the following,

- circle the letters of the ones that are not subspaces
- for any one that is not a subspace, give one of the following reasons why it isn't: (i) it doesn't contain the zero vector, (ii) it is not closed under scalar multiplication, (iii) it is not closed under vector addition

(a) The set of vectors in \mathbb{R}^3 of the form $\begin{bmatrix} 2a \\ b \\ a \end{bmatrix}$.

(b) The set of vectors in \mathbb{R}^3 of the form $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + t \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}$. (i), (ii), (iii)

+1 choices
+1 reasons

(c) The set of vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ in \mathbb{R}^2 for which $xy \leq 0$. (iii)

(d) The xz -plane in \mathbb{R}^3 .

14. Suppose that we are using the least-squares method to find the equation of the line that best approximates the points $(0,1)$, $(1,2)$, $(2,2)$, $(3,4)$. Give the matrix A and the vector \mathbf{b} (label each, of course!) that would be used in the formula $\bar{\mathbf{x}} = (A^T A)^{-1} A^T \mathbf{b}$.

$$y = mx + b$$

$$b = 1$$

$$m + b = 2$$

$$2m + b = 2$$

$$3m + b = 4$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$$

$$\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 4 \end{bmatrix}$$

15. (a) In the space to the right, give the transformation T (not a matrix!) that translates every point in \mathbb{R}^2 three points to the right and one unit down.

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+3 \\ y-1 \end{bmatrix} \quad +2\frac{1}{2}$$

(a) Why can the transformation T not be represented by a 2×2 matrix?

T is not linear. +1

+2 $\frac{1}{2}$

(a) In the space to the right, give the matrix that can be used to perform T when the vectors in \mathbb{R}^2 are put into homogeneous coordinates.

$$[T]_h = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$