Math 341 Homogeneous Coordinates

(1) a) $A\vec{u}_1 = \begin{bmatrix} -1 \\ 22 \end{bmatrix}$, $A\vec{u}_2 = \begin{bmatrix} 23 \\ 4 \end{bmatrix}$, $A\vec{u}_3 = \begin{bmatrix} 6 \\ 21 \end{bmatrix}$

b) $AB = \begin{bmatrix} -1 & 23 & 6 \\ 22 & 4 & 21 \end{bmatrix}$ c) The columns of AB are $A\vec{u}_1, A\vec{u}_2, A\vec{u}_3$

(2) $C\vec{w} = \begin{bmatrix} -1 \\ 22 \\ 1 \end{bmatrix}$

(3) a) $[P'Q'R'] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -2 \\ 2 & 4 & 4 \end{bmatrix}$ b) $\theta = 90°$

(4) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 4 \\ -1 & -1 & -2 \end{bmatrix} = [P'Q'R']$

(6) $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

(7) a) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ c) Yep

(8) a) $[T] = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$, $[T]^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

b) $[T^{-1} R_{90°} T] = [T]^{-1} [R_{90°}] [T] = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 5 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

⑧ c) $\begin{bmatrix} 0 & -1 & 5 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 & 4 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 3 \\ 3 & 5 & 5 \\ 1 & 1 & 1 \end{bmatrix}$ $P'(4,3), Q'(4,5), R'(3,5)$

⑨ a) $$[R_{60°}] = \begin{bmatrix} \cos 60° & -\sin 60° & 0 \\ \sin 60° & \cos 60° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[R_{60°}]^{-1} = [R_{-60°}] = \begin{bmatrix} \cos(-60°) & -\sin(-60°) & 0 \\ \sin(-60°) & \cos(-60°) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

reflection $[B] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

To reflect over the line we first rotate $-60°$, reflect, then rotate $60°$, so we want $[R_{60°}][B][R_{-60°}] = [R_{60°} B R_{-60°}]$. Note that the first transformation is on the right, closest to the vector so that it acts first.

$$[R_{60°} B R_{-60°}] = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} -0.5 & 0.866 & 0 \\ 0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(9) b)
$$\begin{bmatrix} -0.5 & 0.866 & 0 \\ 0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 & 4 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -0.1 & -1.1 & -0.3 \\ 2.2 & 4.0 & 4.5 \\ 1 & 1 & 1 \end{bmatrix}$$

(10) For this we need to translate down by 1 unit, perform the reflection of Exercise 9, then translate up by 1. We translate down 1 with $[T]$ and up 1 with $[T]^{-1}$, where

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } [T]^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$ If $[A]$ is the matrix that does the desired reflection, then

$$[A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A][PQR] = \begin{bmatrix} -0.5 & 0.866 & -0.866 \\ 0.866 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 & 4 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1.0 & -2.0 & -1.1 \\ 2.7 & 4.5 & 5.0 \\ 1 & 1 & 1 \end{bmatrix}$$