

$$\textcircled{1} \begin{bmatrix} 1 & 1 & 1 & 3 & a \\ 2 & 1 & 0 & 3 & b \\ 3 & 1 & 0 & 4 & c \\ 4 & 1 & 1 & 6 & d \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & 1 & \hat{a} \\ 0 & 1 & 0 & 1 & \hat{b} \\ 0 & 0 & 1 & 1 & \hat{c} \\ 0 & 0 & 0 & 0 & \hat{d} \end{bmatrix}$$

The matrices do not span M_{22}

$$\textcircled{2} \text{ a) } c_1 + c_2x + c_3x^2 + c_4(x+x^3) = 2 + 3x - x^2 + 5x^3$$

$$c_1 + (c_2 + c_4)x + c_3x^2 + c_4x^3 = 2 + 3x - x^2 + 5x^3$$

$$c_1 = 2$$

$$c_2 = -2$$

$$c_3 = -1$$

$$c_4 = 5$$

$$\text{b) } c_1 + c_2x^2 + c_3x^3 + c_4(x+x^3) = 2 + 3x - x^2 + 5x^3$$

$$c_1 + c_4x + c_2x^2 + (c_3 + c_4)x^3 = 2 + 3x - x^2 + 5x^3$$

$$c_1 = 2$$

$$c_2 = -1$$

$$c_3 = 2$$

$$c_4 = 3$$

c) See work

$$c_1 + c_2x + c_3x^2 + c_4x^3 + c_5(x+x^3) = 0$$

$$x + x^3 - (x + x^3) = 0$$

$$\text{d) } x^3 + x = 0(1) + 1(x) + 0(x^2) + 1(x^3)$$

For one example

$$\textcircled{4} c_1(x^2 + 2x + 5) + c_2(x^2 - 2x + 1) + c_3(2x^2 + x + 4) = 0$$

$$(c_1 + c_2 + 2c_3)x^2 + (2c_1 - 2c_2 + c_3)x + (5c_1 + c_2 + 4c_3) = 0$$

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 2 & -2 & 1 & 0 \\ 5 & 1 & 4 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The polynomials are linearly independent.

$$(5) \quad c_1 \begin{bmatrix} 5 & 7 \\ 5 & -10 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + c_3 \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix} + c_4 \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} 5c_1 + c_2 + c_4 &= 0 \\ 7c_1 + 2c_2 + 3c_3 + 2c_4 &= 0 \\ 5c_1 + 3c_2 + c_3 &= 0 \\ -10c_1 - 4c_2 + 2c_3 &= 0 \end{aligned} \quad \xrightarrow{\text{rref}} \quad \begin{array}{ccccc} & c_1 & c_2 & c_3 & c_4 \\ \begin{bmatrix} 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 1 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

c_4 is free, so let $c_4 = t$

$$c_3 + \frac{1}{3}c_4 = 0 \implies c_3 = -\frac{1}{3}t$$

$$c_2 - \frac{2}{3}c_4 = 0 \implies c_2 = \frac{2}{3}t$$

$$c_1 + \frac{1}{3}c_4 = 0 \implies c_1 = -\frac{1}{3}t$$

Let $t = 3$ to get $c_1 = -1, c_2 = 2, c_3 = -1, c_4 = 3$

$$-\begin{bmatrix} 5 & 7 \\ 5 & -10 \end{bmatrix} + 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$