Math 342 ASSIGNMENT 22, SPRING 2016 **Due at the start of class on 5/27**

Here is what you should be able to do by the time you finish this assignment:

- Find a linear change of variable that maps one bounded interval to another.
- Use a change of variable to convert an integral on one bounded interval to an integral on any other bounded interval.
- Approximate an integral on $[-1, 1]$ by Gaussian Quadrature, given the roots of Legendre polynomials and the corresponding weights.
- Multiply two 2×2 matrices by hand using the inner product method, showing clearly how it is done.
- Multiply two 2×2 or 3×3 matrices by hand using the outer product method, showing clearly how it is done.
- Determine whether a vector is an eigenvector of a matrix. If it is, determine the corresponding eigenvalue.
- Find eigenvalues and eigenvectors for a 2×2 matrix by hand.

1. Give a linear function $u = mx + b$ that maps

   (a) $[-2, 8]$ to $[-5, 5]$ (b) $[-\frac{1}{3}, \frac{1}{3}]$ to $[-5, 5]$

   (c) $[-5, 1]$ to $[2, 16]$ (d) $[3, 14]$ to $[-2, 2]$

   Give your answers without complex fractions (fractions "inside" fractions). I'd suggest one of the two following methods:

   - Find the equation of a line through two points.
   - Move center to center, then scale.

   I'd also suggest checking to see that endpoints of the first interval convert to the endpoints of the second interval using your equation.

2. Use a linear change of variable to convert the given integral to one on the other interval given. **Show the equation you use to change variables and any algebra you do with it, and show how to get what dx changes to.** Check your answers by computing the original integral and the new one using your calculator or the Wolfram Alpha definite integral widget. (Both integrals should give the same value!)

   (a) $\displaystyle\int_3^9 (x^2 + 5x)\,dx,\quad [-1, 1]$ (b) $\displaystyle\int_{-3}^3 \cos 5x\,dx,\quad [0, 2]$ (c) $\displaystyle\int_2^5 \sqrt{x+1}\,dx,\quad [-1, 1]$

3. Here are the zeros of the first few Legendre polynomials and the corresponding weights for Gaussian quadrature:

   - $n = 2$: $x = \pm\sqrt{\frac{1}{3}},\quad w_1 = 1,\ w_2 = 1$
   - $n = 2$: $x = 0, \pm\sqrt{\frac{3}{5}},\quad w_1 = w_3 = 0.556,\ w_2 = 0.889,$
   - $n = 4$: $x = \pm 0.340,\ \pm 0.861,\quad w_1 = w_4 = 0.348,\ w_2 = w_3 = 0.652.$

   Use these to approximate the following integrals, using the given n:

   (a) $\displaystyle 2\int_{-1}^1 \frac{1}{2u+3}\,du,\quad n = 4$ (b) $\displaystyle 3\int_{-1}^1 \Big[(3u+6)^2 + 5(3u+6)\Big]\,du,\quad n = 3$

   NOTE: Give the integral being approximated followed by $\approx$, then the sum $w_1 f(x_1) + w_2 f(x_2) + \cdots + w_n f(x_n)$ followed by $=$ and your final result, **to the thousandth's place.**

4. For each of $n = 2, 3, 4$ find the sum the weights used for the corresponding Gaussian quadrature. (Add *all* n weights for each value of n). Write two sentences:

- One that indicates what the "question" was, and what your answer is.
- Another that tells why you think the answer is what it is.

5. Let $A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$. Show clearly how to use the inner product method to find AB.

6. Let $A = \begin{bmatrix} 2 & -3 & 0 \\ 1 & 1 & 5 \\ 4 & -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 & 1 \\ -1 & 2 & 6 \\ -4 & 0 & 1 \end{bmatrix}$. Show clearly how to use the outer product method to find AB.

7. Consider the matrix $A = \begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}$ and the vectors $\mathbf{u} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

   (a) Give the vectors $A\mathbf{u}$, $A\mathbf{v}$ and $A\mathbf{w}$. Note that in two of the three cases, multiplying by A was equivalent to just multiplying the vector by a scalar. For which vectors was this the case, and what was the scalar?

   (b) Any *nonzero* vector $\mathbf{x}$ for which there is a scalar λ (lambda) such that $A\mathbf{x} = \lambda\mathbf{x}$ is called an **eigenvector** of the matrix A, with **eigenvalue** λ. Restate your answers to (a) using this language. You should say "______ is an eigenvector of A with eigenvalue ______ ."

   (c) Multiply one of your eigenvectors by three and see if the result is also an eigenvector. If it is, say so and give the corresponding eigenvalue.

   (d) Repeat (c) for the other eigenvector, but multiplying by -2 rather than 3.

   (e) Make a conjecture based on your answers to (c) and (d). Write a sentence giving your conjecture in the conditional form "If ..., then ... ."

8. For each of the following matrices B and C and the given vectors, determine whether any of the vectors are eigenvectors of the matrix. For those that are, say so and give the corresponding eigenvalues.

   (a) $B = \begin{bmatrix} -1 & 0 & 1 \\ 3 & 0 & -3 \\ 1 & 0 & -1 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 5 \\ -15 \\ -5 \end{bmatrix}$

   (b) $C = \begin{bmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

9. Let P be the matrix whose columns are the vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ from Exercise 8(c). Enter P and C in your calculator and compute the product $P^{-1}CP$. Give your answer, labeled as always(!), and then write a sentence summarizing what you observe about $P^{-1}CP$.