(1) $\vec{v}_1 = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

a) $\vec{w}_1 = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}$ b) $\vec{w}_2 = \vec{v}_2 - \text{proj}_{\vec{w}_1}\vec{v}_2 = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} - \overset{+\frac{2}{7}}{\frac{-10}{35}} \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{28}{7} \\ 0 \\ \frac{14}{7} \end{bmatrix} + \begin{bmatrix} -\frac{6}{7} \\ \frac{10}{7} \\ \frac{2}{7} \end{bmatrix} = \begin{bmatrix} \frac{22}{7} \\ \frac{10}{7} \\ \frac{16}{7} \end{bmatrix}$

$\approx \begin{bmatrix} 3.143 \\ 1.429 \\ 2.286 \end{bmatrix}$

c) $\vec{w}_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} - \overset{+\frac{12}{35}}{\frac{-12}{35}} \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} - \frac{\overset{2.571}{(3.143 - 2.858 + 2.286)}}{\underset{17.146}{(3.143^2 + 1.429^2 + 2.286^2)}} \begin{bmatrix} 3.143 \\ 1.429 \\ 2.286 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} -1.029 \\ 1.714 \\ 0.343 \end{bmatrix} - 0.150 \begin{bmatrix} 3.143 \\ 1.429 \\ 2.286 \end{bmatrix} = \begin{bmatrix} -0.500 \\ -0.500 \\ 1.001 \end{bmatrix}$

Parts (d) and (e) on the next page.

(3) $\vec{v}_1 = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ Repeat as above, but with $\langle \vec{u}, \vec{v} \rangle = 3u_1v_1 + u_2v_2 + 2u_3v_3$

$\vec{w}_1 = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}$ $\vec{w}_2 = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} - \frac{\overset{-32}{(-36 + 0 + 4)}}{\underset{54}{(27 + 25 + 2)}} \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} + \frac{16}{27} \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{60}{27} \\ \frac{80}{27} \\ \frac{70}{27} \end{bmatrix}$

Note that this is just a scalar multiple of $\begin{bmatrix} 6 \\ 8 \\ 7 \end{bmatrix}$, which is orthogonal to $\vec{w}_1$ with the given inner product.

We can use $\vec{w}_2 = \begin{bmatrix} 6 \\ 8 \\ 7 \end{bmatrix}$ to find $\vec{w}_3$:

$\vec{w}_3 = \vec{v}_3 - \text{proj}_{\vec{w}_1}\vec{v}_3 - \text{proj}_{\vec{w}_2}\vec{v}_3$

$= \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} - \frac{(-9 - 10 + 2)}{54} \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} - \frac{\overset{16}{(18 - 16 + 14)}}{\underset{270}{(3(36) + 64 + 2(49))}} \begin{bmatrix} 6 \\ 8 \\ 7 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \frac{17}{54} \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} - \frac{16}{270} \begin{bmatrix} 6 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} -0.3 \\ -0.9 \\ 0.9 \end{bmatrix} \Longrightarrow \vec{w}_3 = \begin{bmatrix} -1 \\ -3 \\ 3 \end{bmatrix}$

$$\underbrace{\vec{w}_1 = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}, \vec{w}_2 = \begin{bmatrix} 6 \\ 8 \\ 7 \end{bmatrix}, \vec{w}_3 = \begin{bmatrix} -1 \\ -3 \\ 3 \end{bmatrix}}_{\text{Any scalar multiples of these are OK.}}$$

(1) parts (d) and (e), that I forgot,

(d) $\|\vec{w}_1\| = \sqrt{9+25+1} = \sqrt{35} \Rightarrow \vec{q}_1 = \frac{1}{\sqrt{35}}\begin{bmatrix}-3\\5\\1\end{bmatrix} \approx \begin{bmatrix}-0.507\\0.845\\0.169\end{bmatrix}$

$\|\vec{w}_2\| = \sqrt{(3.143)^2+(1.429)^2+(2.286)^2} = 4.141 \quad \vec{q}_2 = \frac{1}{4.141}\begin{bmatrix}3.143\\1.429\\2.286\end{bmatrix} = \begin{bmatrix}0.759\\0.345\\0.552\end{bmatrix}$

$\|\vec{w}_3\| = \sqrt{(-0.5)^2+(-0.5)^2+(1.001)^2} = 1.226 \quad \vec{q}_3 = \frac{1}{1.226}\begin{bmatrix}-0.5\\-0.5\\1.001\end{bmatrix} = \begin{bmatrix}-0.408\\-0.408\\0.816\end{bmatrix}$

(e) $\begin{bmatrix}-0.507 & 0.845 & 0.169\\0.759 & 0.345 & 0.552\\-0.408 & -0.408 & 0.816\end{bmatrix} \cdot \begin{bmatrix}-0.507 & 0.759 & -0.408\\0.845 & 0.345 & -0.408\\0.169 & 0.552 & 0.816\end{bmatrix} \approx \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

3.5:

(2) a) $\|f\|_1 = \frac{1}{2}\int_{-1}^{1} |x^3+x^2-\frac{1}{2}|dx = 0.4469$

$\|f\|_2 = \left(\frac{1}{2}\int_{-1}^{1}(x^3+x^2-\frac{1}{2})^2 dx\right)^{\frac{1}{2}} = 0.5094$

b) $\|f\|_\infty = \max_{x\in[-1,1]} \{|f(x)|\} = ?$

(3) a) $f'(x) = 3x^2+2x = x(3x+2)$ $f'(x)=0$ when $x=0, -\frac{2}{3}$

$f(0) = -\frac{1}{2}$, $f(-\frac{2}{3}) = (-\frac{2}{3})^3 + (-\frac{2}{3})^2 - \frac{1}{2} = -\frac{8}{27} + \frac{4}{9} - \frac{1}{2} = -\frac{16}{54} + \frac{24}{54} - \frac{27}{54}$

$= -\frac{19}{54}$

b) $f(-1) = -\frac{1}{2}$ $f(1) = \frac{3}{2}$

c) $\|f\|_\infty = \frac{3}{2}$

(4) $f(x) = xe^{-x}$ $f'(x) = -xe^{-x}+e^{-x} = e^{-x}(-x+1)$

$f'(x)=0$ when $x=1$. $f(1) = e^{-1}$, $f(0)=0$, $f(2) = 2e^{-2}$

≈ 0.3679 ≈ 0.2707

(7) $\|e^x-(0.54x^2+1.10x+1)\|_1 = \frac{1}{2}\int_{-1}^{1} |e^x-0.54x^2-1.10x-1|dx =$

$\|e^x-(0.54x^2+1.10x+1)\|_2 = \left(\frac{1}{2}\int(e^x-0.54x^2-1.10x-1)^2 dx\right)^{\frac{1}{2}} =$

$\|e^x-(0.54x^2+1.10x+1)\|_\infty =$