1. Find the norm of $\mathbf{u} = \begin{bmatrix} -5 \\ 2 \\ 3 \end{bmatrix}$ using the norm induced on $\mathbb{R}^3$ by $\langle \mathbf{u}, \mathbf{v} \rangle = 3u_1v_1 + u_2v_2 + 2u_3v_3$.

2. Sketch the unit circle for each of the following norms on $\mathbb{R}^2$:

   (a) $\|\mathbf{x}\| = \max\left\{ |\frac{1}{2}x_1|, |\frac{1}{3}x_2| \right\}$ (b) $\|\mathbf{x}\| = \sqrt{\frac{1}{16}x_1^2 + \frac{1}{9}x_2^2}$

3. Find the distance between $f(x) = x$ and $g(x) = \sin x$ on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ with the distances induced by the 1-, 2- and $\infty$-norms, as defined on page 66 of the book.

4. The vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ form a basis for $\mathbb{R}^3$.

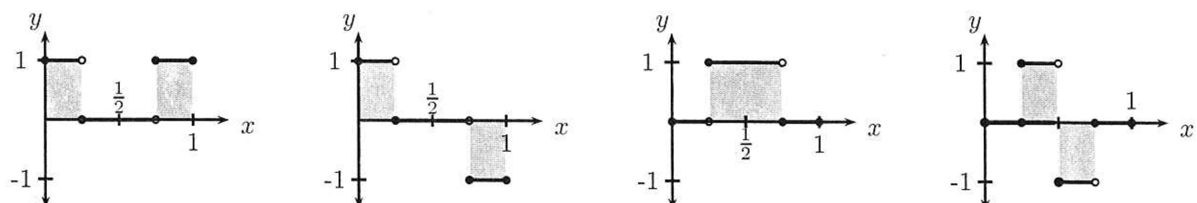
   (a) Apply the Gram-Schmidt process to find the first two vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ of an orthogonal basis.

   (b) The components of $\mathbf{w}_2$ are fractions. Multiply that vector by a scalar in order to eliminate the fractions - the new vector will still be orthogonal to $\mathbf{w}_1$. Make an abuse of notation and call the new, scaled vector $\mathbf{w}_2$ also.

   (c) Continue applying the Gram-Schmidt process to obtain the third vector of the orthogonal basis. Scale it as well so that all vectors in the orthogonal basis have integer components.

5. Let $Q$ be an orthogonal matrix, which means that each of its columns is orthogonal to all the other columns. Describe, as specifically as you can, what the matrix $Q^TQ$ looks like.

6. The set $\mathcal{C}$ of functions below is an orthogonal basis for the type of step functions on $[0, 1]$ that we worked with on the signal processing assignments.



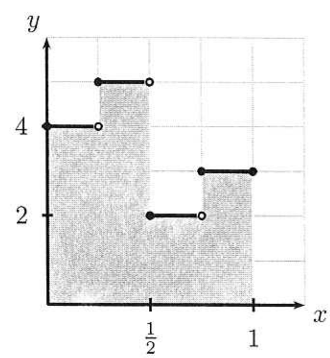

$c_1(x)$ $c_2(x)$ $c_3(x)$ $c_4(x)$

   (a) For the step function $f$ whose graph is shown to the right, give the coordinate vector $[f]_{\mathcal{C}}$, the coordinate vector with respect to $\mathcal{C}$.

   (b) Let $\mathcal{E}$ be the usual standard basis. Find the change of basis matrix $[P]_{\mathcal{E},\mathcal{C}}$.

   (c) Find $\langle f, c_2 \rangle$ and $\langle c_2, c_2 \rangle$ using the standard integral inner product on $[0, 1]$.

   (d) What does the value $\dfrac{\langle f, c_2 \rangle}{\langle c_2, c_2 \rangle}$ represent?




7. For the set $\mathscr{C}[0, \infty)$, define an inner product by $\langle f, g \rangle = \displaystyle\int_0^\infty f(x)\, g(x)\, e^{-x}\, dx$. Perform the Gram-Schmidt process on the first three elements of the set $\mathcal{B} = \{1, x, x^2, x^3, ...\}$. **Give all coefficients in exact form.** Use the Wolfram Alpha definite integral widget if need be. Wolfram recognizes the letters *inf* as $\infty$.