① Is there a linear combination of

$$2\begin{bmatrix}-1\\-7\\3\\11\end{bmatrix} - 5\cdot\begin{bmatrix}1\\-3\\2\\5\end{bmatrix} + 1\begin{bmatrix}7\\-1\\4\\3\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$y'' - 3y' + 2y = 0 \qquad y = 0$

Example 1.4(c)

$S = \{-1 + x + 2x^2, 1 + x + x^2, -3 + x + 3x^2\}$

$S \subseteq \mathcal{P}_2$ Does S span $\mathcal{P}_2$?

Can every $p(x)$ in $\mathcal{P}_2$ be written as a linear combination of $-1+x+2x^2, 1+x+x^2, -3+x+3x^2$?

$$ax^2+bx+c = c_1(-1+x+2x^2)+c_2(\quad)+c_3(\quad)$$

$$-c_1+c_2-3c_3 = c$$
$$= b$$
$$= a$$

$$\longrightarrow \begin{bmatrix} 0 & \begin{matrix} a \\ b \\ c \end{matrix} \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 2 & \hat{a} \\ 0 & 1 & -1 & \hat{b} \\ 0 & 0 & 0 & \hat{c} \end{bmatrix}$$

rref $0c_1+0c_2+0c_3=\hat{c}$

Suppose $\vec{v} = c_1\vec{u}_1 + c_2\vec{u}_2 + c_3\vec{u}_3 + c_4\vec{u}_4$

and $\boxed{\vec{u}_4 = d_1\vec{u}_1 + d_2\vec{u}_2 + d_3\vec{u}_3} \Rightarrow u_3 = \frac{\vec{u}_4 - d_1 u_1 \cdots}{d_3}$

$+ u_4 = 0$

$$\vec{v} = c_1\vec{u}_1 + c_2\vec{u}_2 + c_3 u_3 + c_4\left(d_1\vec{u}_1 + d_2 u_2 + d_3\vec{u}_3\right)$$

$$\vec{v} = (c_1 + c_4 d_1)\vec{u}_1 + (c_2 + c_4 d_2)\vec{u}_2 + \cdots$$

☆ $\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4$ are linearly dependent

$V_1 = c_2 V_2 + c_3$

$V_2 =$

$$\begin{bmatrix} -1 & 1 & 7 & 0 \\ & \sim & & \\ & \sim & & \\ & \sim & & \end{bmatrix} \xrightarrow{rref} \begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array}$$

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \rightarrow c_1 - 2c_3 = 0 \\ \rightarrow c_2 + 5c_3 = 0 \\ \rightarrow 0c_1 + 0c_2 + 0c_3 = 0 \\ \\ \end{array}$$

c_3 is free, let $c_3 = t$

$c_2 = -5t$

$c_1 = 2t$

$$\begin{array}{cccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \\ \end{array}$$

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 & 5 & 1 & 0 \\ 0 & 0 & 0 & 1 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_2 = r$

$x_4 = 3s - 2t$

$x_5 = s$

$x_6 = t$

② Repeat ① for $\begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 7 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ -1 \end{bmatrix}$

$$\underline{c_1}\begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix} + \underline{c_2}\begin{bmatrix} 4 \\ 7 \\ 0 \end{bmatrix} + \underline{c_3}\begin{bmatrix} -2 \\ 5 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-3c_1 + 4c_2 - 2c_3 = 0$$

$$\sim\sim\sim = 0$$

$$\sim\sim\sim = 0$$