numbers x vectors $\vec{x}$

functions f matrices

① Vector spaces

② Linear transformations

$$T(\vec{v}) = T(c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3) = c_1T(\vec{v}_1) + c_2T(\vec{v}_2) + c_3T(\vec{v}_3)$$

"Vectors"
transformations
(transforms,
operators)

2.1: 3,4,5,7 Write out like the examples, can be less wordy + more sequential

Let $T: P_2 \to P_1$ be defined by $T(ax^2+bx+c) = bx+a$

① What is $T(-x^2+5x-3)$?

② Is $T[p(x)+q(x)] = T(p(x)) + T[q(x)]$? for all $p,q \in P_2$

Let $T: M_{22} \to \mathbb{R}$ be defined by

$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad$

③ Is $T(cA) = cT(A)$ for all $c \in \mathbb{R}, A \in M_{22}$?

④ Is $T(A+B) = T(A) + T(B)$ for all $A, B \in M_{22}$?

Let $p, q \in P_2$.

Then $p(x) = a_1x^2 + b_1x + c_1$, $q(x) = a_2x^2 + b_2x + c_2$

$$T[p(x)+q(x)] = T[a_1x^2+b_1x+c_1+a_2x^2+b_2x+c_2]$$

$$= T[(a_1+a_2)x^2+(b_1+b_2)x+(c_1+c_2)]$$

$$= (b_1+b_2)x + (a_1+a_2)$$

$$= b_1x+b_2x+a_1+a_2$$

$$= (b_1x+a_1)+(b_2x+a_2)$$

$$= T[a_1x^2+b_1x+c_1]+T[a_2x^2+b_2x+c_2]$$

$$= T[p(x)] + T[q(x)]$$

$p(x) = a_1x^2 + b_1x + c_1$

$q(x) = a_2x^2 + b_2x + c_2$

If $c=\pi$ and $A=\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}$, then

$T(cA)=T\left(\pi\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}\right)=\cdots\cdots=4\pi^2$

$cT(A)=\pi T\left(\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}\right)=4\pi\neq 4\pi^2=T(cA)$

$\sqrt{x}$

$\sqrt{a+b} \stackrel{?}{\neq} \sqrt{a} + \sqrt{b}$

$\sin(a+b) \neq \sin a + \sin b$

$A(\vec{u}+\vec{v}) = A\vec{u} + A\vec{v}$