$T: \mathcal{P}_2 \longrightarrow \mathcal{P}_2$ is defined by

$$[T(p)](x) = x^2 p''(x)$$

$p(x) = 3x^2 + 7x - 4$ $\quad [T(p)](x) = 6x^2$

$p'(x) = 6x + 7$

$p''(x) = 6$

Let

$p(x) = a_1 x^2 + a_2 x + a_3$

$q(x) = b_1 x^2 + b_2 x + b_3$

$T(p+q) = T(a_1 x^2 + b_1 x^2 + a_2 x + b_2 x + a_3 + b_3)$

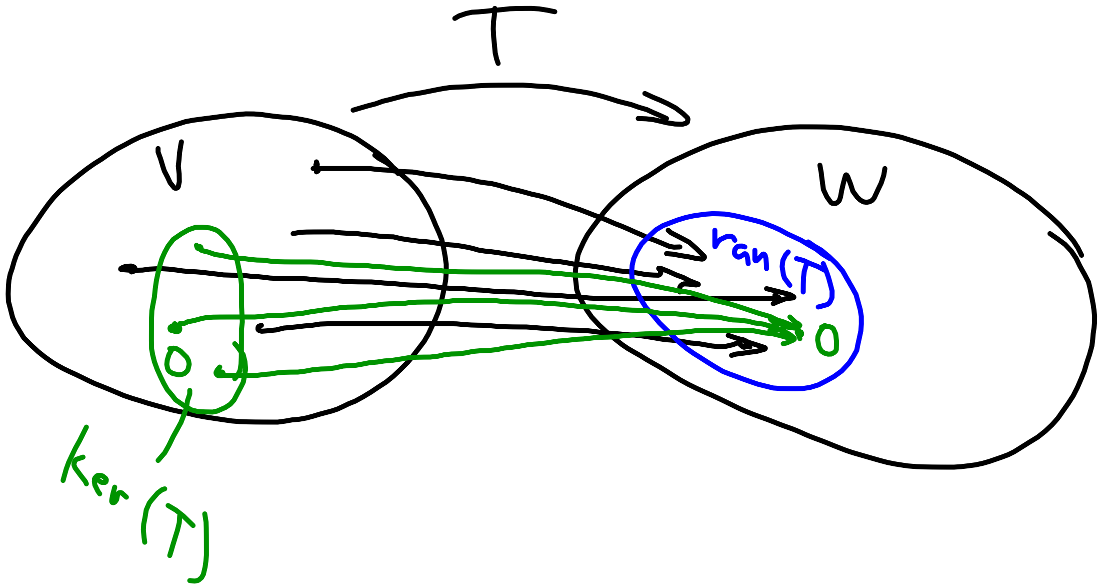
$p'' = 2a_1$ $\quad = x^2(2a_1 + 2b_1) = x^2(2a_1) + x^2(2b_1)$

$q'' = 2b_1$ $\quad = x^2 p'' + x^2 q''$

$T(p+q) = x^2(2a_1 + 2b_1)$ $\quad = T(p) + T(q)$

$(p+q)'' = 2a_1 + 2b_1$

Therefore it meets the first condition of linearity.


T
V
W
ran(T)
0
0
ker(T)

$T: \mathcal{P}_2 \to \mathcal{P}_2 \quad [T(p)](x) = x^2 p''(x)$

$T(0x^2 + 2x + 7) = 0$

$T(0x^2 + bx + c) = 0$

$\ker(T) = \{ bx + c \mid b \text{ and } c \in \mathbb{R} \} = \mathcal{P}_1$

① $\operatorname{ran}(T) = ?$

② Basis for $\ker(T)$, basis for $\operatorname{ran}(T)$

$T: \mathcal{P}_2 \to \mathcal{P}_2$ defined by $[T(p)](x) = x^2 p''(x)$

V

$\mathcal{B}_{\ker(T)} = \{x, 1\}$

~~The~~ basis for $\operatorname{ran}(T)$ is $\{x^2\}$

A

$$T: M_{22} \longrightarrow \mathbb{R} \qquad T(A) = \text{trace}(A)$$

$$\begin{bmatrix} 0 & \\ & 0 \end{bmatrix} \quad \begin{bmatrix} 17 & 0 \\ 0 & 0 \end{bmatrix} \quad 17$$

$$\begin{bmatrix} 2 & 7 \\ -\pi & -2 \end{bmatrix} \in \ker T$$

$$\begin{bmatrix} a & b \\ c & -a \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix} + \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}$$

$\text{nullity}(T) = 3$

$\text{nullity}(T) + \text{rank}(T) = 4$

$3 + 1 =$

$\dim(\ker(T)) = 2 =$ the nullity of T

$\dim(\text{ran}(T)) = \underline{1} = \text{rank}$

<u>Rank Thm</u>

$\text{rank}(T) + \text{nullity}(T) = \dim(V)$