1 Note that the diff eqn $\boxed{y''+25y=0}$ can be rearranged to get $D(y)=0$ $y''=-25y$. Find as many solutions as you can.

Find a function y whose second derivative is -25 times the original function

$y = \sin 5x$

$y = \cos 5x$

$y = C_1 \sin 5x + C_2 \cos 5x$

$y = \boxed{e^{5ix}} = \cos 5x + i \sin 5x$

$y = \boxed{e^{-5ix}} = \cos 5x - i \sin 5x$

$\sin(-5x) = -\sin(5x)$

$D = \frac{d^2}{dx^2} + 25$

$$D = \frac{d^2}{dx^2} + 6\frac{d}{dx} + 9$$

$$D(y) = \frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y$$

$$y'' + 6y' + 9y = 0$$

guess $y = e^{rx}$

$$e^{rx}(r^2 + 6r + 9) = 0$$

$$e^{rx}(r+3)(r+3) = 0$$

$$r = -3$$

$y = Ce^{-3x}$ is in $\ker(D)$

$$y = C_1e^{-3x} + C_2xe^{-3x}$$

② Note that $\lambda^2 > 0$ if $\lambda \neq 0$. Find as many solutions to $\boxed{y'' + \lambda^2 y = 0}$ as you can.

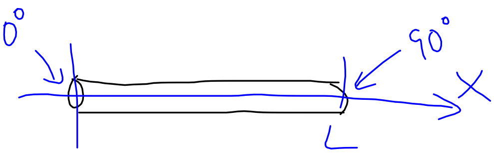
$y = \sin \lambda x$

$y = \cos \lambda x$

In general,

$y = C_1 \sin \lambda x + C_2 \cos \lambda x$ is the general solution to

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$ Heat equation



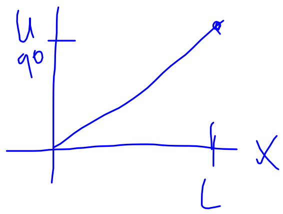
$$\frac{X(x)T'(t)}{XTk} = \frac{kX''(x)T(t)}{XTk}$$

$$-\lambda^2 = \frac{1}{k}\frac{T'}{T} = \frac{X''}{X}$$

Find: u(x,t) is temp at x, at time t.

Assume $u(x,t) = X(x)T(t)$

$\cancel{x+t}$



$$X'' + \lambda^2 X = 0 \qquad T' + k\lambda^2 T = 0$$

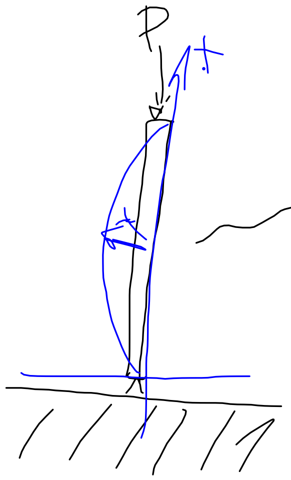
$$D_x = \frac{d^2}{dx^2} + \lambda^2 \qquad D_t = \frac{d}{dt} + k\lambda^2$$

$$u = XT$$

$$e^{-k\lambda^2 n t}$$

$\sin n\lambda x$

$\cos n\lambda x$



Diff eqn $\quad \dfrac{\cancel{EI}}{\cancel{EI}} \dfrac{d^2y}{dx^2} = -\dfrac{P}{EI} y$

From properties of materials
E, I, P constant

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = 0$$

What now?

* A little more w/ transformations
* Orthogonality
* More LA-DE connections

$f(x) = x + 1$

$g(x) = x^2 - 3x$

$(g \circ f)(x) = g[f(x)] = g[x+1] = (x+1)^2 - 3(x+1)$

$= x^2 - x - 2 \quad (?)$

Notation

$f, g \in \mathcal{F}$ (or any C^n)

$p, q \in \mathcal{P}_n$ $\qquad \vec{u}, \vec{v} \in \mathbb{R}^n$

$A, B \in M_{22}$

S, T are transformations

$\mathcal{S}$ is a set, as are $\mathcal{B}, \mathcal{C}$ (bases)