Proving one-to-one (1-1)

$T: V \to W$ $\quad (f: A \to B)$

Let $\vec{u}, \vec{v} \in V$ such that $T(\vec{u}) = T(\vec{v})$.
Show that $\vec{u} = \vec{v}$.

$T: M_{22} \to M_{22}$ $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} d & c \\ b & a \end{bmatrix}$

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and

$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \in M_{22}$ such that

$T(A) = T(B)$.

$$T(A) = \begin{bmatrix} a_{22} & a_{21} \\ a_{12} & a_{11} \end{bmatrix} \quad T(B) = \begin{bmatrix} b_{22} & b_{21} \\ b_{12} & b_{11} \end{bmatrix}$$

Since $T(A) = T(B)$, $a_{11} = b_{11}$

$a_{12} = b_{12}$

$a_{21} = b_{21}$

$a_{22} = b_{22}$

$A = B$ since each element is equal.

$\therefore$ $T(A) = T(B)$ implies $A = B$.

$\therefore$ T is one-one.

$T: V \to W$ is 1-1 if, and only if, (iff)

$P \Longleftrightarrow Q$

$T(\vec{u}) = T(\vec{v})$ implies $\vec{u} = \vec{v}$.

(If $T(\vec{u}) = T(\vec{v})$, then $\vec{u} = \vec{v}$.)

$p \to q$

$\sim(p \to q)$

$p \wedge \sim q$

$T: V \to W$ is invertible

if there is an $S: W \to V$

such that

$(S \circ T)(\vec{v}) = \vec{v}$ 2.3: S

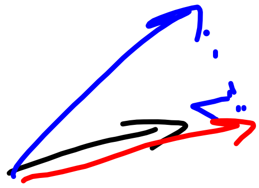
and $(T \circ S)(\vec{v}) = \vec{v}$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\left[\, A \mid I \,\right] \xrightarrow{\text{rref}} \left[\, I \mid B \,\right]$$

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$



$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

$\vec{u} \cdot \vec{v} = 0$ iff $\vec{u}$ is perp to $\vec{v}$

$$\|\vec{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2} = \sqrt{\vec{u} \cdot \vec{u}}$$

Dot product gives us projections.

$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ can be thought of

as a function $u : \{1,2,3\} \longrightarrow \mathbb{R}$

Example: $\vec{u} = \begin{bmatrix} -7 \\ 5 \\ 4 \end{bmatrix}$

$\underset{u_1}{u(1)} = -7$ $\underset{u_2}{u(2)} = 5$ $\underset{u_3}{u(3)} = 4$

$$\vec{u} = \begin{bmatrix} -7 \\ 5 \\ 4 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 6 \\ 8 \\ -5 \end{bmatrix}$$

$$\vec{u} \cdot \vec{v} = (-7)(6) + (5)(8) + (4)(-5)$$

$$= u(1)v(1) + u(2)v(2) + u(3)v(3)$$

$$= \sum_{k=1}^{3} u(k)v(k)$$

$$\langle f, g \rangle = \int_a^b f(x)g(x)\,dx$$

inner product

For Wed

1. Prove T from 2.1:5 is 1-1
2. 2.3:4
3. 3.1:2