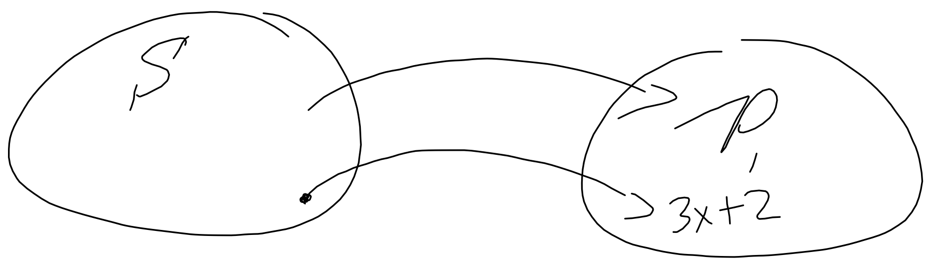
Let $D = \frac{d}{dx}$ $\qquad C^1 \xrightarrow{D} C^1$

① Is $D : C^1 \to C^1$ invertible? If so, give $D^{-1}(ax+b)$.

② Let $S = \{ax^2 + bx \mid a, b \in \mathbb{R}\}$ and consider $D : S \to \mathcal{P}_1$. Is $\underline{\underline{D}}$ invertible? If so, what is $D^{-1}(ax+b) = \frac{a}{2}x^2 + bx$


S
P
3x+2

$T: \mathcal{P}_1 \to \mathcal{P}_1$ defined by $T(ax+b) = 2bx - a$

Find the inverse T^{-1}.

Want $T^{-1}(ax+b) = cx+d$, but with c, d in terms of $a + b$

$T(cx+d) = ax+b$ (should be)

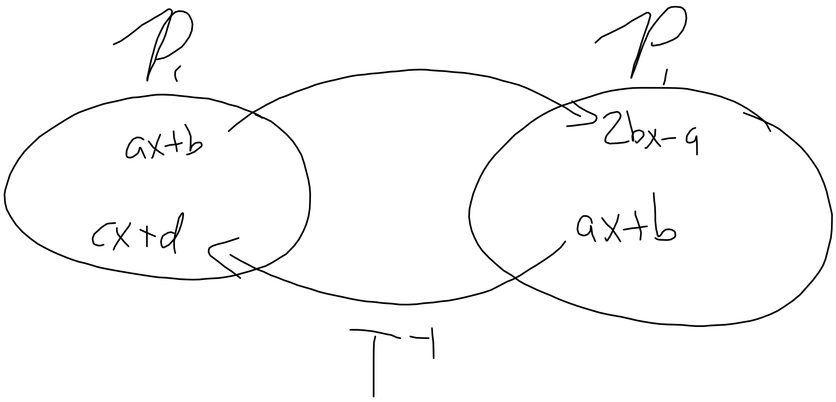
$T(cx+d) = 2dx - c$

So $2dx - c = ax+b$

$2d = a \qquad -c = b$

$d = \frac{1}{2}a \qquad c = -b$

$\to T^{-1}(ax+b) = -bx + \frac{1}{2}a$


P1
ax+b
cx+d
P1
2bx-a
ax+b
T-1

Solve $x_1 - x_2 - x_3 + 2x_4 = $

$$\Longrightarrow \vec{x} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

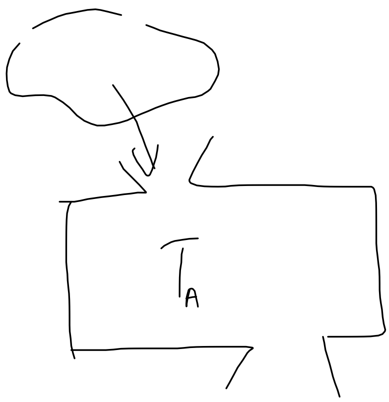
$$\underbrace{\begin{bmatrix} 1 & -1 & -1 & 2 \\ 2 & -2 & -1 & 3 \\ -1 & 1 & -1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 1 \\ 3 \\ -3 \end{bmatrix}}_{\vec{b}}$$

$$T(A) = A\vec{x}$$

$$\mathbb{R}^4 \to \mathbb{R}^3$$

$$A(\vec{u} + \vec{v}) = A\vec{u} + A\vec{v}$$

$$A(c\vec{u}) = cA\vec{u}$$



nullity $= 2$

$\dim(V) = \dim(\mathbb{R}^4) = 4$

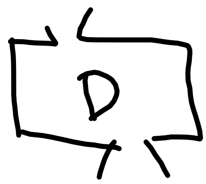
$\Longrightarrow \operatorname{rank}(T_A) = 2$

$$\cancel{D =} \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 5\sin 3x$$

$$D(e^{rx}) = 0 \text{ if } r = -1, -3$$

$$\ker(D) = \{C_1 e^{-x} + C_2 e^{-3x}\}$$

homogeneous sol



$$D(y) = \underbrace{5\sin 3x}_{\text{in ran}(D)} \text{ when } y = -\tfrac{1}{6}\sin 3x - \tfrac{1}{3}\cos 3x$$

$$\boxed{y = \quad + C_1 e^{-x} + C_2 e^{-3x}}$$

$$\langle \vec{u}, \vec{v} \rangle =$$

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\langle \vec{u}, \vec{v} \rangle = \vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2$$

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$