Exam

1. Subspace?
   * If yes, give a basis
   * if no, reason/counterexample
2. Linearly dep or indep?
3. Find a linear combination.
4. Anything else we've done.

$$S = \left\{ \begin{bmatrix} a \\ a^2 \\ a^3 \end{bmatrix} \middle| \, a \in \mathbb{R} \right\}$$

Not a subspace.

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix} \in S. \quad \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 9 \end{bmatrix} \notin S$$

S is not closed under addition, so it is not a subspace.

$$\begin{bmatrix} a \\ 3a \\ b \end{bmatrix} = \begin{bmatrix} a \\ 3a \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix}$$

$$= a\begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The set of vectors like this is the span of $\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$, so it is a subspace

$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathcal{B}_1 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$ax^2 + bx + c = a(x^2) + b(x) + c(1)$$

$\mathcal{B} = \{x^2, x, 1\}$ is a basis for $\mathcal{P}_2$

$$\mathcal{B} = \{x+1, x^2+1, x^2+x\}$$

$$\mathcal{B} = \{x^2, x^2+x, x^2+x+1\}$$

$$\mathcal{B} = \{1, 1+x, 1+x+x^2\}$$

$S = \{1+x, x+x^2, 1+2x+x_2, x^2-1\}$

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\} = \mathcal{B}_1$$

$$\underset{A}{\overset{}{\begin{bmatrix} 3 & -2 \\ 0 & 5 \end{bmatrix}}} = 3\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - 2\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 5\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[A]_{\mathcal{B}_1} = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$$

$$\mathcal{B}_2 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \right\}$$

$$\underset{A}{\begin{bmatrix} 3 & -2 \\ 0 & 5 \end{bmatrix}} = 5\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + (-7)\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$[A]_{\mathcal{B}_2} = \begin{bmatrix} 5 \\ -7 \\ 5 \end{bmatrix}$$