$$S = \left\{ \begin{bmatrix} a \\ b \\ a+b \end{bmatrix} \middle| a, b \in \mathbb{R} \right\} \subseteq \mathbb{R}^3$$

$$\begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} \pi \\ e \\ e+\pi \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \in S$$

$$\begin{bmatrix} 7 \\ 7 \\ 7 \end{bmatrix} \notin S$$

$$\begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 11 \end{bmatrix} \in S$$

$$\begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 9 \end{bmatrix} \in S$$

$$3\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \\ 3 \end{bmatrix} \in S$$

Prove $S = \left\{ \begin{bmatrix} a \\ b \\ a+b \end{bmatrix} \right\}$ is a subspace of $\mathbb{R}^3$.

Need to show that S is closed under

* addition * scalar mult

WTS if $\vec{u}, \vec{v} \in S$, then $\vec{u} + \vec{v} \in S$

Let $\vec{u}, \vec{v} \in S$ hypothesis/assumption

$$\vec{u} = \begin{bmatrix} a_1 \\ b_1 \\ a_1+b_1 \end{bmatrix}, \vec{v} = \begin{bmatrix} a_2 \\ b_2 \\ a_2+b_2 \end{bmatrix}$$ def of S

$$\vec{u} + \vec{v} = \begin{bmatrix} a_1 \\ b_1 \\ a_1+b_1 \end{bmatrix} + \begin{bmatrix} a_2 \\ b_2 \\ a_2+b_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 \\ b_1+b_2 \\ (a_1+a_2)+(b_1+b_2) \end{bmatrix} \in S$$ algebra

$\vec{u}+\vec{v} \in S$, so S is closed under addition. conclusion

Let $\vec{u} \in S$, $c \in \mathbb{R}$

$$\vec{u} = \begin{bmatrix} a \\ b \\ a+b \end{bmatrix}$$

$$c\vec{u} = \ldots \begin{bmatrix} ca \\ cb \\ ca+cb \end{bmatrix} \in S$$

Henceforth S is closed under scalar multiplication, and S is a subspace of $\mathbb{R}^3$

Show that the even functions are a subspace of $\mathcal{F}$.

Let f and g be even.

$f(x)+g(x)$

$(f+g)(x)$

$f(-x)=f(x)$, $g(-x)=g(x)$

$(f+g)(-x)=f(-x)+g(-x)=f(x)+g(x)=(f+g)(x)$

def of $f+g$ $\qquad$ f,g are even

f is even

(cf)

$f+g$ is even

The even functions are closed under addition.

$$(cf)(-x) = c\left[f(-x)\right] = c\left[f(x)\right] = (cf)(x)$$

$$S' = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \in \mathbb{R}^2 \,\middle|\, ab \geq 0 \right\}$$

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix}, \underbrace{\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -5 \end{bmatrix}}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix} \notin S'$$

$\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -5 \end{bmatrix} \in S$, but $\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -1 \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $(1)(-2) \ngeq 0$

so $\begin{bmatrix} 1 \\ -2 \end{bmatrix} \notin S$ and S not closed under addition. S is not subspace.

$$S = \left\{ \begin{bmatrix} a & b \\ b & a \end{bmatrix} \middle| a, b \in \mathbb{R} \right\} \subseteq M_{22}$$

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} + \begin{bmatrix} 0 & b \\ b & 0 \end{bmatrix}$$

$$= a \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

For Monday

1.3: 1b, d, e, 5, 12, 13 Write a general element as a lin comb of specific "vectors".

1.3: 1b, 16 prove subspaces

1.3: 1c give a specific counterexample