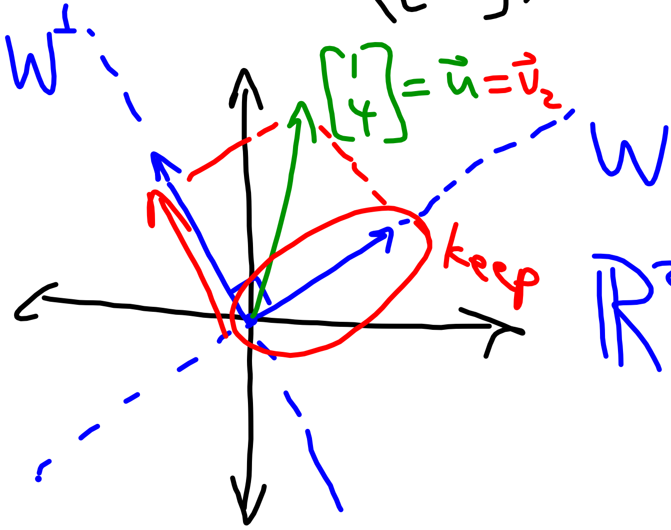
3.3:1

$W = \text{span}\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix}\right)$

$\vec{v} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $\vec{v}_1$

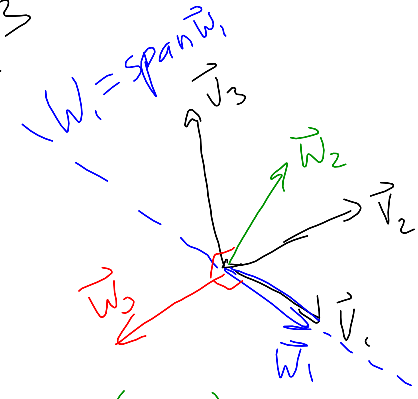


$W^{\perp} = \text{span}\left(\begin{bmatrix} -2 \\ 3 \end{bmatrix}\right)$

$\mathbb{R}^2 = W \oplus W^{\perp}$

direct sum

$\mathbb{R}^3$



$W_2 = \text{span}(\vec{w}_1, \vec{w}_2)$

Create orthogonal basis

$\mathcal{B} = \{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$

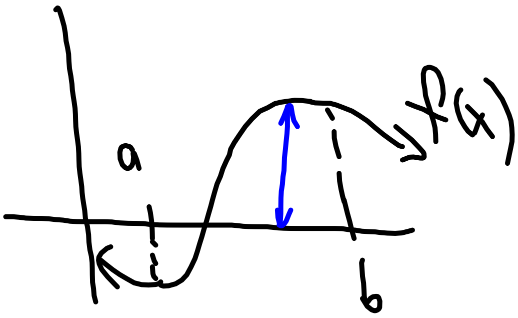
$\vec{w}_1 = \vec{v}_1$

$\vec{w}_2 = \vec{v}_2 - \text{proj}_{W_1} \vec{v}_2$

$\vec{w}_3 = \vec{v}_3 - \text{proj}_{W_2} \vec{v}_3 = \vec{v}_3 - \text{proj}_{\vec{w}_1} \vec{v}_3 - \text{proj}_{\vec{w}_2} \vec{v}_3$

Norm (magnitude, length)

way of giving the 'size' of a vector



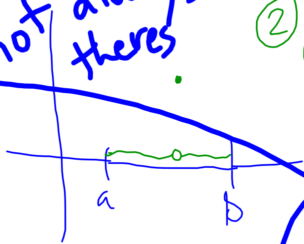
How big is f on $[a,b]$?

Options

$$\|f\|_\infty = \max\{|f(x)| \mid x \in [a,b]\}$$

① Largest value of $|f(x)|$ on $[a,b]$

Problem: not always there!



② Get out of this by averaging

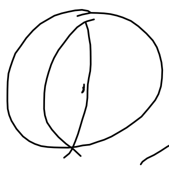
$$\frac{1}{b-a}\int_a^b |f(x)|\,dx = \|f\|_1$$

"1-norm"

③ $$\|f\|_2 = \sqrt{\frac{1}{b-a}\int_a^b [f(x)]^2\,dx}$$

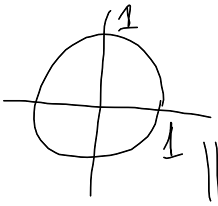
Natural with $\langle f,g\rangle = \frac{1}{b-a}\int_a^b f(x)g(x)\,dx$

$$\|f\|_p = \left(\frac{1}{b-a}\int_a^b |f(x)|^p dx\right)^{\frac{1}{p}}$$



$1 \le p < \infty$

Vector $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ $\quad \|\vec{u}\|_2 = \sqrt{u_1^2 + u_2^2 + u_3^2} = \sqrt{\cancel{3}\sum_{n=1}^{3} u_n^2}$



$\|\vec{u}\|_1 = |u_1| + |u_2| + |u_3|$

$\|\vec{u}\|_p = \left(|u_1|^p + |u_2|^p\right)^{\frac{1}{p}}$

$\|\vec{u}\|_1 = \max\{|u_1|, |u_2|, |u_3|\}$