$$\underbrace{\vec{v}_1 = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}}_{\vec{w}_1}, \vec{v}_2 = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\vec{w}_2 = \underbrace{\vec{v}_2 - \text{proj}_{\vec{w}_1}\vec{v}_2}_{\text{perp}_{\vec{w}_1}\vec{v}_2} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} + \frac{\overset{2}{\cancel{-10}}}{\underset{7}{\cancel{35}}} \frac{\cancel{\vec{v}_2 \cdot \vec{w}_1}}{\cancel{\vec{w}_1 \cdot \vec{w}_1}} \vec{w}_1 = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} -\frac{6}{7} \\ \frac{10}{7} \\ \frac{2}{7} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{22}{7} \\ \frac{10}{7} \\ \frac{16}{7} \end{bmatrix} = \vec{w}_2$$

$$\text{new } \vec{W}_2 = \cancel{\begin{bmatrix} 22 \\ 10 \\ 16 \end{bmatrix}} \begin{bmatrix} 11 \\ 5 \\ 8 \end{bmatrix}$$

$1, x, x^2, x^3, x^4, \ldots \qquad [-1, 1]$

$\int_{-1}^{1}$

$$\vec{W}_3 = \vec{V}_3 - \text{proj}_{\vec{W}_1} v_3 - \text{proj}_{\vec{W}_2} v_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \frac{\overset{72}{\cancel{\vec{V}_3 \cdot \vec{W}_1}}}{\underset{210}{\cancel{\vec{W}_1 \cdot \vec{W}_1}}} \vec{W}_1 - \frac{\overset{9}{\cancel{\vec{V}_3 \cdot \vec{W}_2}}}{\underset{210}{\cancel{\vec{W}_2 \cdot \vec{W}_2}}} \vec{W}_2$$

($\cancel{+12}$ 72, $\cancel{35}$ 210)

$$\vec{W}_1 = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}$$

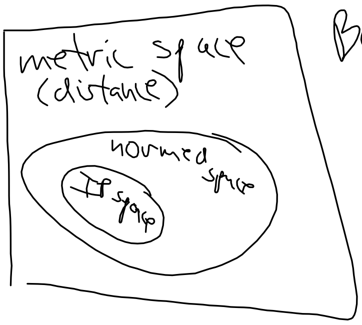
$$= \begin{bmatrix} \frac{210}{210} \\ \frac{-420}{210} \\ \frac{210}{210} \end{bmatrix} + \begin{bmatrix} \frac{-216}{210} \\ \frac{360}{210} \\ \frac{72}{210} \end{bmatrix} - \begin{bmatrix} \frac{99}{210} \\ \frac{45}{210} \\ \frac{72}{210} \end{bmatrix}$$

$$\vec{W}_2 = \begin{bmatrix} 11 \\ 5 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{93}{210} \\ \frac{-105}{210} \\ \frac{210}{210} \end{bmatrix} = \begin{bmatrix} \frac{31}{70} \\ -\frac{1}{2} \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 31 \\ -35 \\ 70 \end{bmatrix} = \vec{W}_3$$

$$\vec{W}_3 = \begin{bmatrix} \cancel{31} \ -35 \\ -35 \\ 70 \end{bmatrix}$$

Distance, norm, inner product



Banach

$d(x,y)$ distance

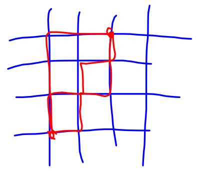
norm induces a distance $d(x,y) = \|x-y\|$

inner product induces a norm

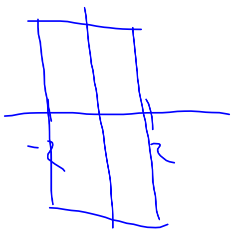
$\|\vec{u}\| = \sqrt{\langle \vec{u}, \vec{u} \rangle}$

$\|\vec{x}\| = \max\left\{\left|\frac{1}{2}x_1\right|, \left|\frac{1}{3}x_2\right|\right\}$,

$\vec{x} = \begin{bmatrix}1\\3\end{bmatrix}$

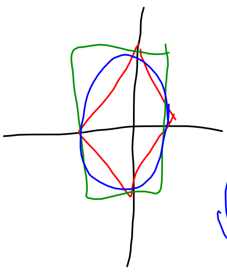


$x_1 = 2$, $x_1 = -2$ with $-3 \le x_2 \le 3$



$\|\vec{x}\| = \max\left\{\left|\frac{1}{2}(1)\right|, \left|\frac{1}{3}(3)\right|\right\}$

$= 1$



$\sqrt{x_1^2 + x_2^2}$

$|x_1| + |x_2|$