

A linear transformation $T: V \rightarrow W$ is invertible if, and only if, T is 1-1 and onto.

A linear transformation T is 1-1 if, and only if $\ker(T) = 0$.

A linear transformation T is onto if, and only if, $\text{ran}(T) = W$.

T is linear

$T: M_{22} \rightarrow \mathcal{P}_3$, $\text{rank}(T) = 4$. Is T invertible? If not, why not?

$T: M_{22} \rightarrow \mathcal{P}_2$ " "

$T: \mathbb{R}^3 \rightarrow \mathcal{P}_2$, $\text{nullity}(T) = 1$ " "

3.1 1c) $\langle \vec{u}, \vec{v} \rangle = u_1 + v_2$ $\langle \vec{u} + \vec{v}, \vec{w} \rangle = \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle$?

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \vec{v} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}, \vec{w} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\left\langle \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \end{bmatrix} \right\rangle = \left\langle \begin{bmatrix} -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \end{bmatrix} \right\rangle = 5$$

A is an $n \times n$ matrix that...

Then $\langle \vec{u}, \vec{v} \rangle$ defined on \mathbb{R}^n by

$\vec{u}^T A \vec{v}$ is an inner product.

$$[u_1, u_2] \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$\textcircled{5} \quad A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \quad \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix} \implies \langle \vec{u}, \vec{v} \rangle \neq \langle \vec{v}, \vec{u} \rangle$$

a) 31 $A^T = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} = A$ A is symmetric

b) $\sqrt{50}$

c) Find a vector \vec{w} that is orthogonal to $\vec{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

A relation R is an equivalence relation if

- ① xRx reflexive
- ② If xRy , then yRx symmetric
- ③ If xRy and yRz , then xRz transitive

$$\text{Want } \vec{w} \mid \langle \vec{u}, \vec{w} \rangle = 0 \quad \vec{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$\begin{aligned} \langle \vec{u}, \vec{w} \rangle &= \begin{bmatrix} 3 & -2 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \\ &= \begin{bmatrix} 11 & 4 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \end{aligned}$$

$$= 11w_1 + 4w_2$$

$$11w_1 + 4w_2 = 0$$

$$w_1 = 1$$

$$4w_2 = -11$$

$$w_2 = \frac{-11}{4}$$

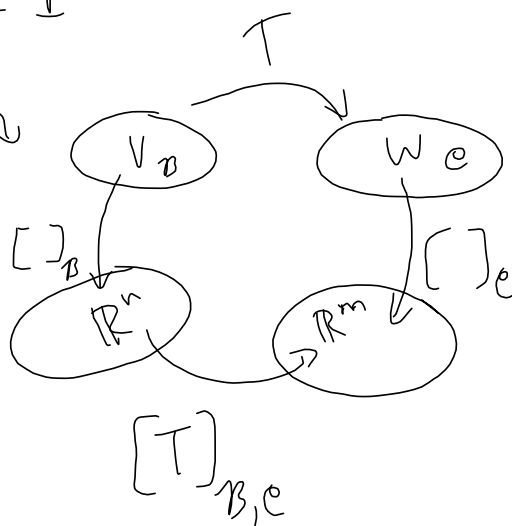
$$\left\langle \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \end{bmatrix} \right\rangle + \left\langle \begin{bmatrix} -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \end{bmatrix} \right\rangle = 8 + 4 = 12$$

For Wed

3.2:6

2.4:1

$$\begin{aligned} & \vec{v} \cdot \vec{v} \\ & \langle \vec{v}, \vec{v} \rangle \end{aligned}$$



[]