A linear transformation $T: V \to W$ is invertible if, and only if, T is 1-1 and onto.

A linear transformation T is 1-1 if, and only if $\ker(T) = 0$.

A linear transformation T is onto if, and only if, $\operatorname{ran}(T) = W$.

T is linear

$T: M_{22} \to P_3$, rank$(T)=4$. Is T invertible? If not, why not?

$T: M_{22} \to P_2$ " "

$T: \mathbb{R}^3 \to P_2$, nullity$(T)=1$ " "

3.1 1c) $\langle \vec{u}, \vec{v} \rangle = u_1 + v_2$

$\langle \vec{u}+\vec{v}, \vec{w} \rangle \stackrel{?}{=} \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle$

$\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \vec{v} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}, \vec{w} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$\left\langle \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \end{bmatrix} \right\rangle = \left\langle \begin{bmatrix} -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \end{bmatrix} \right\rangle = 5$

A is an $n \times n$ matrix that...

Then $\langle \vec{u}, \vec{v} \rangle$ defined on $\mathbb{R}^n$ by

$\vec{u}^T A \vec{v}$ is an inner product.

$$[u_1, u_2] \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

5 $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$ $\begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix} \Longrightarrow \langle \vec{u}, \vec{v} \rangle \neq \langle \vec{v}, \vec{u} \rangle$

a) 31 $A^T = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} = A$ A is symmetric

b) $\sqrt{50}$

c) Find a vector $\vec{w}$ that is orthogonal to $\vec{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

A relation R is an equivalence relation if

1. xRx reflexive
2. If xRy, then yRx symmetric
3. If xRy and yRz, then xRz transitive

Want $\vec{w} \mid \langle \vec{u}, \vec{w} \rangle = 0 \quad \vec{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

$$\langle \vec{u}, \vec{w} \rangle = \begin{bmatrix} 3 & -2 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 4 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$= 11w_1 + 4w_2$$

$11w_1 + 4w_2 = 0$

$w_1 = 1$

$4w_2 = -11$

$w_2 = \frac{-11}{4}$

$$\left\langle \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \end{bmatrix} \right\rangle + \left\langle \begin{bmatrix} -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \end{bmatrix} \right\rangle = 8 + 4 = 12$$

For Wed 3.2:6

2.4:1

$\sqrt{\vec{v}\cdot\vec{v}}$

$\sqrt{\langle \vec{v},\vec{v}\rangle}$

$$\begin{tikzcd}
V_{\mathcal{B}} \arrow[r, bend left, "T"] \arrow[d, "{[\ ]_{\mathcal{B}}}"'] & W_{\mathcal{C}} \arrow[d, "{[\ ]_{\mathcal{C}}}"] \\
\mathbb{R}^n \arrow[r, bend right, "{[T]_{\mathcal{B},\mathcal{C}}}"'] & \mathbb{R}^m
\end{tikzcd}$$

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