

① Finish filling out the table:
(or values)

② Find a value of r such that $y = e^{rt}$ is a solution to

$\frac{d^2y}{dt^2}$

$$y'' + 8y' + 15y = 0 \quad \therefore \dots$$

$$y = C_1 e^{-3t} + C_2 e^{-5t}$$

n	a_n	$\frac{n^2}{2}$
1	2	1 $\downarrow +2$
2	6	3 $\downarrow +3$
3	12	6 $\downarrow +4$
4	20	10
5	30	15
10	110	$\frac{n(n+1)}{2}$
n	$n(n+1)$	Explicit

$$C_1 e^{-3t} + C_2 e^{-5t} = 0$$

$$C_1 e^{2t} + C_2 = 0$$

$$C_1 e^{2t} = -C_2$$

$$e^{2t} = -\frac{C_2}{C_1}$$

③ Find a value or values of p such that $y = x^p$ is a solution to $x^2 y'' + 2xy' - 6y = 0$.

$$p = -3, 2$$

$$y = C_1 x^{-3} + C_2 x^2$$

$$u(r, \theta)$$

$$u(r, \theta) = \underline{\underline{R(r)}} \underline{\underline{\Theta(\theta)}}$$

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0$$

$$\text{Suppose } y = a_0 + a_1x + a_2x^2 + \dots$$

$$\begin{aligned}6a_0 + 2a_2 &= 0 \\2a_2 &= -6a_0 \\a_2 &= -3a_0\end{aligned}$$

$$\begin{aligned}12a_4 &= 0 \\a_4 &= 0 \\a_6 &= 0 \\&\vdots\end{aligned}$$

$$\begin{aligned}4a_1 + 6a_3 &= 0 \\6a_3 &= -4a_1 \\a_3 &= -\frac{2}{3}a_1\end{aligned}$$

$$\begin{aligned}-6a_3 + 20a_5 &= 0 \\20a_5 &= 6a_3 \\a_5 &= \frac{3}{10}a_3 = \frac{3}{10}\left(-\frac{2}{3}a_1\right) \\&= -\frac{1}{5}a_1\end{aligned}$$

Zeros

$$n=0 \quad P_0(x)=1 \quad \text{no zeros}$$

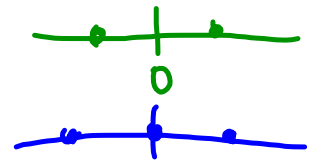
$$n=1 \quad P_1(x)=x \quad x=0$$

$$n=2 \quad x = \pm \sqrt{\frac{1}{3}}$$

$$n=3 \quad x = 0, \pm \sqrt{\frac{3}{5}}$$

$$n=4$$

$$x = \pm 0.340, \pm 0.861$$



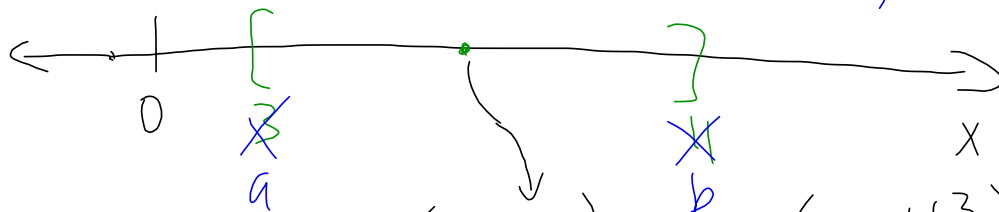
$$\int_{-1}^1 \underbrace{\cos x}_{f(x)} dx \approx \sum_{k=1}^n w_k f(x_k) \stackrel{n=3}{=} 0.556 \cos\left(-\sqrt{\frac{3}{5}}\right) + 0.889 \underbrace{\cos(0)}_{\substack{\text{zeros of} \\ P_n \text{ for a} \\ \text{fixed } n}} + 0.556 \cos\left(\sqrt{\frac{3}{5}}\right)$$

$w_1 = w_3 = 0.556, w_2 = 0.889$

$$[3, 11] \longrightarrow [-1, 1]$$

$$u(x) = \text{circled}(x, a, b)$$

$$[a, b] \rightarrow [-1, 1]$$



$$\frac{\left(x - \frac{11+3}{2}\right)}{\frac{11-3}{2}} = \frac{1}{4} \left(x - \frac{11+3}{2}\right) = u(x)$$

$$\int_{x=2}^7 x \sin x dx = \int_{u=-1}^1 u$$

$$u = \frac{2}{7-2} \left(x - \frac{7+2}{2} \right)$$

$$u = \frac{2}{5} \left(x - \frac{9}{2} \right) \quad \text{When } x=2, u = \frac{2}{5}(2) - \frac{9}{5} = \frac{-5}{5} = -1$$

$$u = \frac{2}{5}x - \frac{9}{5}$$

