① Legendre 1-14 due today

② " 15-20 due tomorrow (Wed)

③ Assn 20+21 due Friday

LU factorization

least-squares

LU factorization

$U = \begin{bmatrix} 2 & 3 & 6 \\ 0 & 5 & 4 \\ 0 & 0 & -1 \end{bmatrix}$ $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

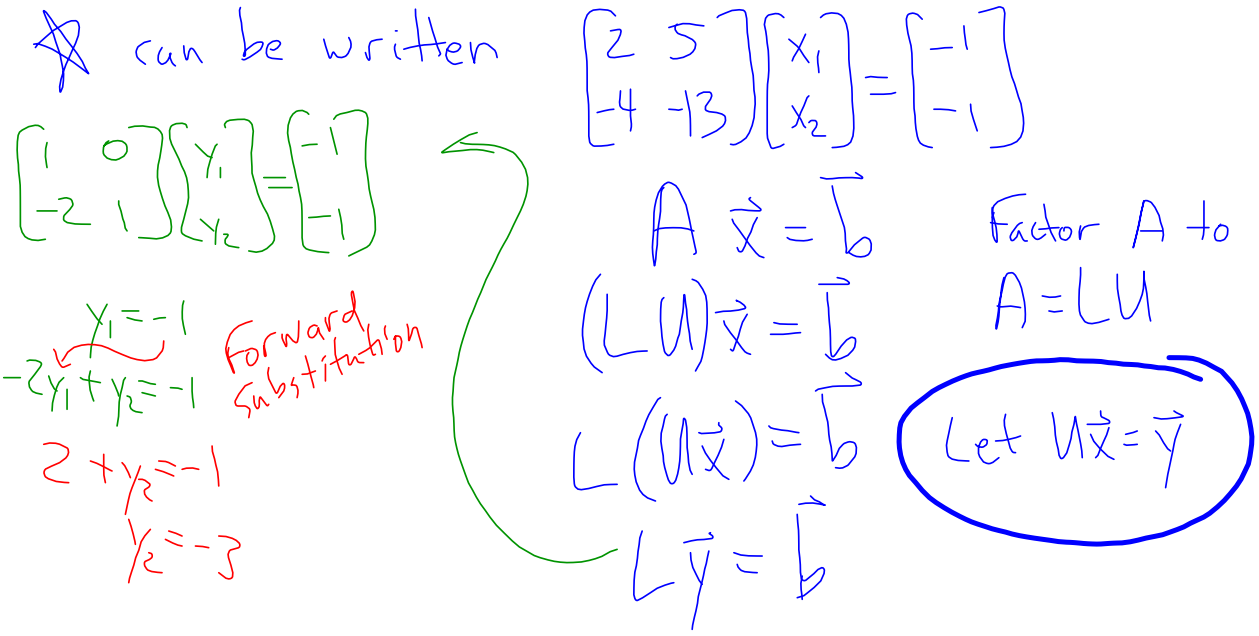
$L = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 2 & 5 \\ 0 & -3 \end{bmatrix}$ $A = LU = \begin{bmatrix} 2 & 5 \\ -4 & -13 \end{bmatrix}$

Solve ★

$$2x_1 + 5x_2 = -1$$
$$-4x_1 - 13x_2 = -1$$

Start here

$A\vec{x} = \begin{bmatrix} 2 & 5 \\ -4 & -13 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$


can be written
$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}=\begin{bmatrix} -1 \\ -1 \end{bmatrix}$
$y_1=-1$
$-2y_1+y_2=-1$
Forward substitution
$2+y_2=-1$
$y_2=-3$
$\begin{bmatrix} 2 & 5 \\ -4 & -13 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} -1 \\ -1 \end{bmatrix}$
$A\vec{x}=\vec{b}$
$(LU)\vec{x}=\vec{b}$
$L((U\vec{x})=\vec{b}$
$L\vec{y}=\vec{b}$
Factor A to
$A=LU$
Let $U\vec{x}=\vec{y}$

$U\vec{x}=\vec{y}$

$$\begin{bmatrix} 2 & 5 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} -1 \\ -3 \end{bmatrix}$$

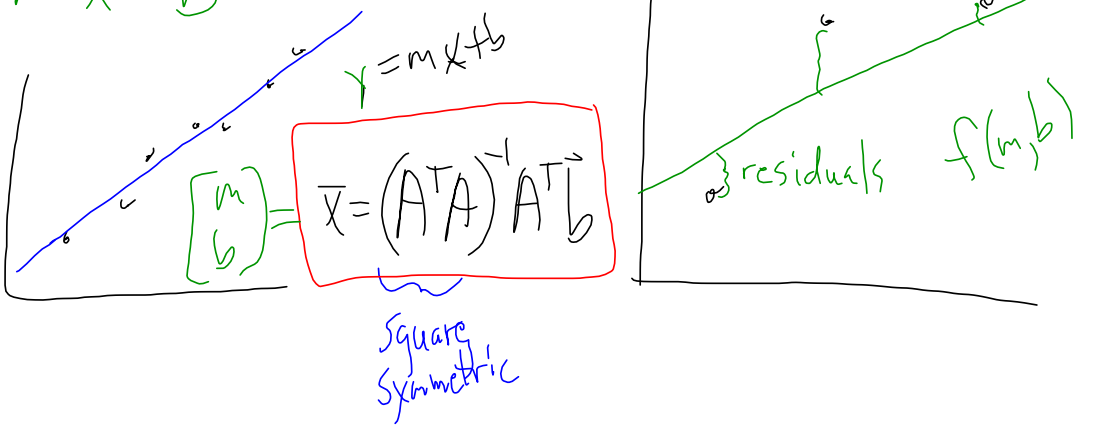
$2x_1+5x_2=-1 \longrightarrow 2x_1+5=-1$

$-3x_2=-3 \qquad 2x_1=-6$

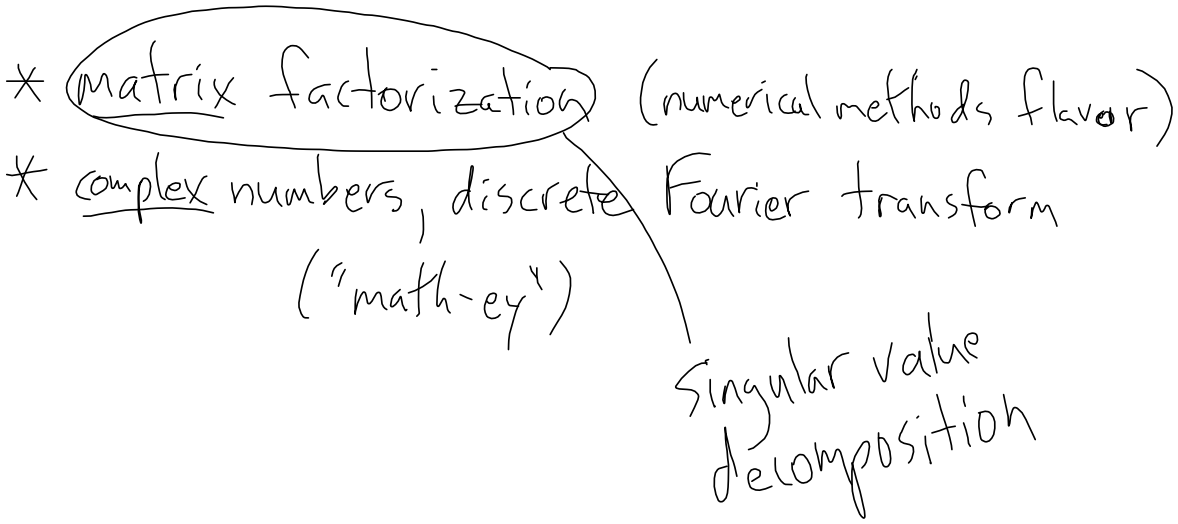
$x_2=1 \qquad x_1=-3$

Least squares

$A\vec{x} = \vec{b}$ → $\vec{x} = A^{-1}\vec{b}$



Square
Symmetric


* matrix factorization (numerical methods flavor)
* complex numbers, discrete Fourier transform ("math-ey")
Singular value decomposition

#14 $\int_2^3 x \sin x\,dx = \int_{-1}^{1} u\,du$

$u = \frac{2}{5}x - \frac{9}{5}$

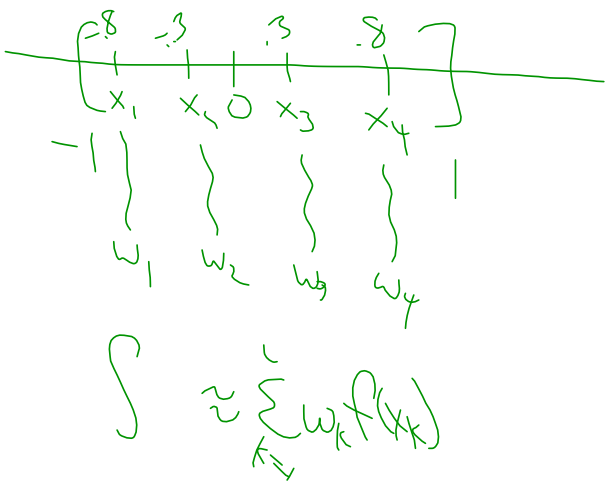
solve for x, put

Find dx in terms of du

$u = \dfrac{x - \frac{a+b}{2}}{\frac{b-a}{2}}$

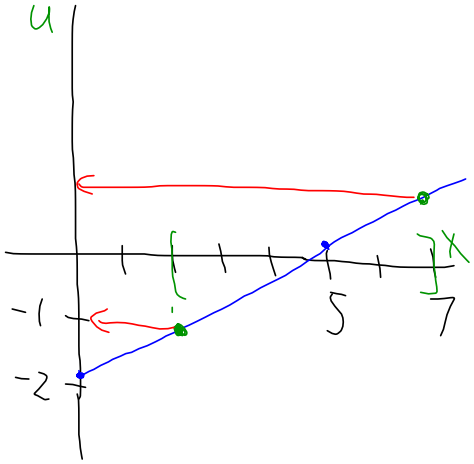
$u = \frac{2}{b-a}\left(x - \frac{a+b}{2}\right)$

$u = \frac{2}{b-a}x - \frac{a+b}{b-a}$



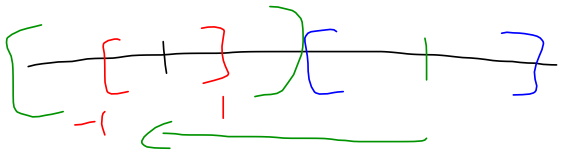
$$\int \approx \sum_{k=1}^{4} w_k f(x_k)$$

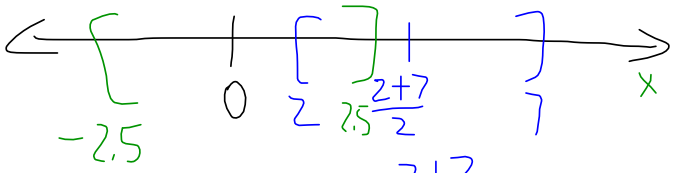
$$u = \frac{2}{5}x - \frac{9}{5}$$



$$\frac{b-a}{2}u = x - \frac{a+b}{2}$$

$$\frac{b-a}{2}u + \frac{a+b}{2} = x$$





$$t = \frac{x - \frac{2+7}{2}}{\frac{7-2}{2}}$$