Multiply $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ without a calculator?

$$\begin{array}{cc} A & B \end{array}$$

Inner Product Method

$$AB = \begin{bmatrix} [1\ 2]\begin{bmatrix}5\\7\end{bmatrix} & [1\ 2]\begin{bmatrix}6\\8\end{bmatrix} \\ [3\ 4]\begin{bmatrix}5\\7\end{bmatrix} & [3\ 4]\begin{bmatrix}6\\8\end{bmatrix} \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

Every row of A "times" every column of B.

Outer product method

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}\begin{bmatrix} 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \end{bmatrix}\begin{bmatrix} 7 & 8 \end{bmatrix}$$

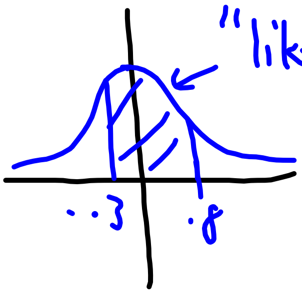
Each column of A "times" the corresponding row of B!

$$= \begin{bmatrix} 5 & 6 \\ 15 & 18 \end{bmatrix} + \begin{bmatrix} 14 & 16 \\ 28 & 32 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

① Why numerical integration?

$$\ln(7) = \int_1^7 \frac{1}{x}\,dx = \ln x\Big|_1^7 = \ln(7) - \ln(1)$$

??



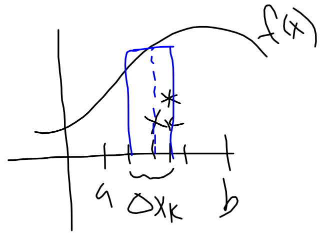
"like" $y=e^{-x^2}$

$$\int_{-.3}^{.8} e^{-x^2}\,dx$$

No antiderivate!

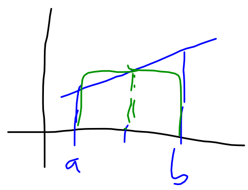
② How does integration work?

$$\int_a^b f(x)\,dx = \lim \sum_{k=1}^{n} f(x_k^*)\,\Delta x_k$$



Then maybe, if we do x_k^* and Δx_k "right"

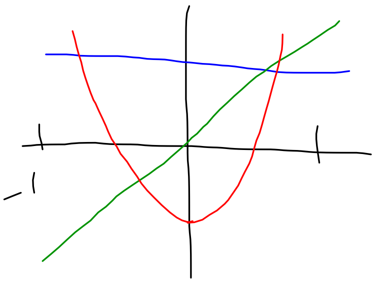
$$\int_a^b f(x)\,dx \approx \sum_{k=1}^{n} f(x_k^*)\,\Delta x_k$$



③ Enter Gaussian quadrature!

$$\int_{-1}^{1} f(x)\,dx \approx \sum w_k f(x_k)$$

Where the x_k's are special points between -1 and 1, and w_k's are the magic numbers that make the $\approx$ good!



(4) Once we can integrate on $[-1,1]$, we can integrate on any $[a,b]$.

$[5,\infty)$ Laguerre

Hermite $(-\infty,\infty)$

Aside

$$\int_{x=1}^{5}(3x-1)^2\,dx = \int_{u=2}^{14}\left[3\left(\frac{u+1}{3}\right)-1\right]^2 \frac{1}{3}\,du = \frac{1}{3}\int_{2}^{14} u^2\,du$$

$$\boxed{u=3x-1} \longrightarrow u+1=3x$$

$$du=3\,dx \longrightarrow dx=\frac{1}{3}\,du \qquad \boxed{\frac{u+1}{3}=x}$$

when $x=1,\ u=2$

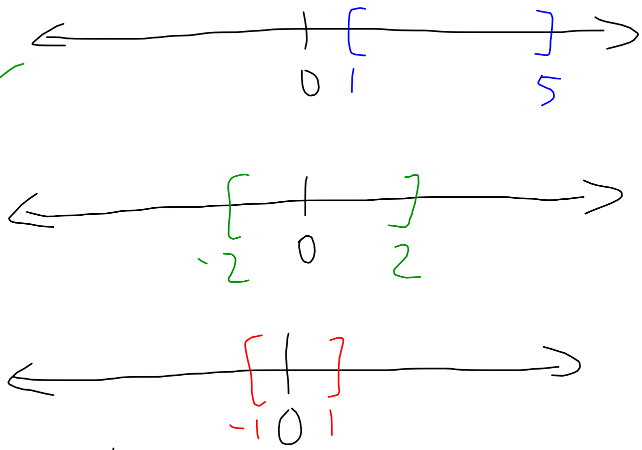
$x=5,\ u=14$

$$\int \omega = \frac{5}{\pi}\int_{\frac{\pi}{4}}$$

$$=\frac{5}{4}$$

(5) Changing interval $\quad \int_1^5 (3x-1)^2\,dx$

Goal: $\int_{-1}^{1}$



$x \longmapsto x-3 \qquad \frac{a+b}{2}$

$x \longmapsto \frac{x}{2}$

$$y = \frac{\left(x - \underset{}{\overset{\frac{a+b}{2}}{3}}\right)}{\underset{\frac{b-a}{2}}{2}}$$

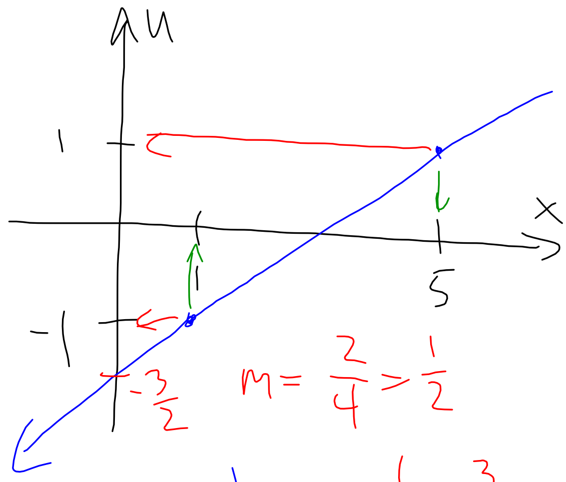
In general,

$$y = \frac{2}{b-a}\left(x - \frac{a+b}{2}\right) = \frac{2}{b-a}x - \frac{a+b}{a-b}$$

Alternately

want $u = 1$ when $x = 5$

$u = -1$ when $x = 1$



$u = mx + b \quad u = \frac{1}{2}x - \frac{3}{2}$

$$\int_{2}^{7} x \sin x \, dx = \int_{-1}^{1} u \sin u \, du$$

$\sin(7)$

$\sin(2)$