Let $f(x)=\frac{3}{2}x^2-\frac{1}{2}$. Use Gaussian quadrature with $x_1=-\sqrt{\frac{3}{5}}$, $x_2=0$, $x_3=\sqrt{\frac{3}{5}}$ and just the letters $\underline{w_1, w_2, w_3}$ to approximate

$$\int_{-1}^{1}\left(\frac{3}{2}x^2-\frac{1}{2}\right)dx \overset{\text{exact}}{=} 0$$

Chosen so that the method is exact for all P_n with $n<3$.

$$0=\underbrace{\left[\frac{3}{2}\left(-\sqrt{\frac{3}{5}}\right)^2-\frac{1}{2}\right]}_{}\overset{f(x_1)}{}w_1+\left(-\frac{1}{2}\right)w_2+\underbrace{\quad}_{}w_3$$

$$\frac{2}{5}w_1 - \frac{1}{2}w_2 + \frac{2}{5}w_3 = 0 \qquad n=2 \longrightarrow 4w_1 - 5w_2 + 4w_3 = 0$$

$$w_1 + w_2 + w_3 = 2 \qquad n=0$$

$$-\sqrt{\frac{3}{5}}w_1 + \cancel{0w_2} + \sqrt{\frac{3}{5}}w_3 = 0 \qquad n=1$$

$$w_1 = w_3$$

$$8w_1 - 5w_2 = 0$$

$$2w_1 + w_2 = 2$$

$$18w_1 = 10$$

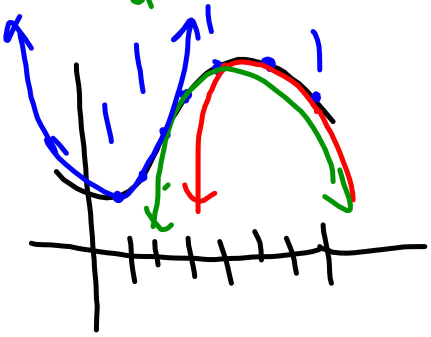
$$w_1 = \frac{10}{18} = \frac{5}{9}$$

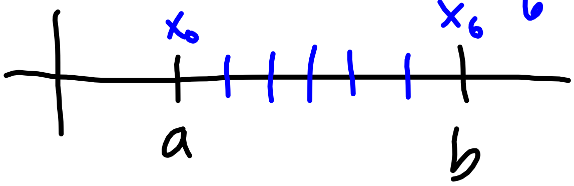
$$w_3 = \frac{5}{9}$$

Simpson's Rule (n even)

n: # of intervals

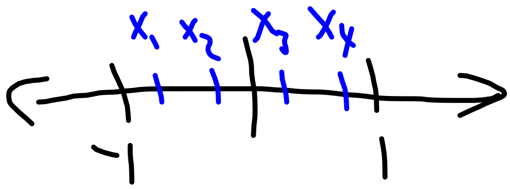
$$\int_a^b f(x)\,dx \approx$$





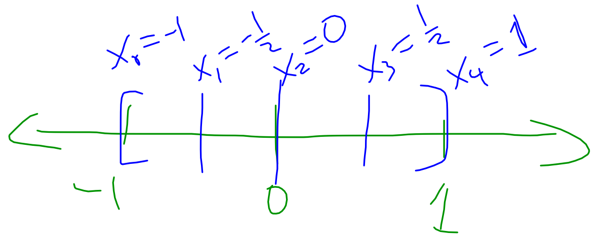
6 intervals

$$\sum_{k=0}^{6} w_k f(x_k)$$

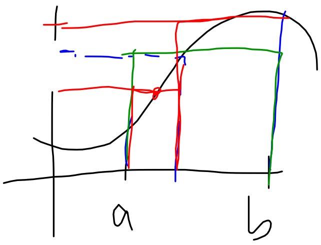


$$\int_a^b f(x)\,dx = \frac{b-a}{3n}\left(f(x_0)+4f(x_1)+2f(x_2)+\cdots \quad 4f(x_{n-1})+f(x_n)\right)$$

$n=4$ on $[-1,1]$:



$$\frac{b-a}{12}\left(f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+f(x_4)\right)$$



$$\frac{b-a}{12} + \frac{4(b-a)}{12} + \frac{2(b-a)}{12} + \frac{4(b-a)}{12} + \frac{b-a}{12}$$

$$\frac{\cancel{12}(b-a)}{\cancel{12}}$$

$$A\vec{x} = \lambda\vec{x}$$

$$A(c\vec{x}) = cA\vec{x} = c\lambda\vec{x} = \lambda(c\vec{x})$$

$$\begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \end{bmatrix}$$

Numerical Integration

A method is exact for a function f if...