$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$ orthogal matrix

Find A^TA and B^TB

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$ orthonormal matrix

$$[A|I] \xRightarrow{\text{vref}} [I|B]$$

$$B = A^{-1}$$

Thm. 3.6 + 3.8 Vector space V

3.6 $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ is orthogonal $\vec{u}$

$$\vec{u} = \frac{\langle \vec{u}, \vec{v}_1 \rangle}{\langle \vec{v}_1, \vec{v}_1 \rangle} \vec{v}_1 + \frac{\langle \vec{u}, \vec{v}_2 \rangle}{\langle \vec{v}_2, \vec{v}_2 \rangle} \vec{v}_2 + \cdots$$

3.8 $\mathcal{B}$ is orthonormal (orthoganal, each $\|\vec{v}_i\| = 1$)

$$\frac{\int_{-1}^{1} x \sin x \, dx}{\int_{-1}^{1} x^2 \, dx}$$

$$\int_0^{2\pi} \sin nx \cos mx \, dx = 0$$

$$\int_0^{2\pi} \sin nx \sin mx \, dx = 0 \text{ if } m \neq n$$

$$\mathcal{B} = \{1, \cos x, \cos 2x, \cos 3x, \ldots, \sin x, \sin 2x, \ldots\}$$

$$\mathcal{B} = \{1, x, x^2, x^3, x^4, \ldots\}$$

$$A^T A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

orthogonal $\longrightarrow$ independent

$$B^T B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B^{-1} = B^T$$