differential equation

$x' = -2x$, initial condition $x(0) = 5$ IVP

$x = Ce^{-2t}$

$5 = Ce^{-2(0)}$ → 1

$x = 5e^{-2t}$

$x_1' = x_1 + 2x_2$

$x_2' = 3x_1 + 2x_2$

$x_1(0) = 10$

$x_2(0) = 5$

Let $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

system of diff eqns

Solve

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\vec{x}' = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \vec{x}$$

Assume $\vec{x} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} e^{kt}$. Then $\vec{x}' = k \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} e^{kt}$

$$k \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} e^{kt} = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} e^{kt}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = k \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$A \quad \vec{c} = k\vec{c}$$

$$A\vec{x} = \lambda\vec{x}$$

$k_1 = 4, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $k_2 = -1, \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}(0) = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ Given IC

Solution to the system is $\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} e^{4t}, \vec{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t}$

General solution is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = d_1 \begin{bmatrix} 2 \\ 3 \end{bmatrix} e^{4t} + d_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t}$

Applying IC, $\begin{bmatrix}10\\5\end{bmatrix} = d_1\begin{bmatrix}2\\3\end{bmatrix} + d_2\begin{bmatrix}-1\\1\end{bmatrix}$

$10 = 2d_1 - d_2$
$5 = 3d_1 + d_2$

$15 = 5d_1$
$3 = d_1$
$d_2 = -4$

$\vec{x} = 3\begin{bmatrix}2\\3\end{bmatrix}e^{4t} - 4\begin{bmatrix}-1\\1\end{bmatrix}e^{-t}$

$x_1 = 6e^{4t} + 4e^{-t}$
$x_2 = 9e^{4t} - 4e^{-t}$

$x_1' = x_1 + 2x_2$
$x_2' = 3x_1 + 2x_2$

Final Thurs 10-12

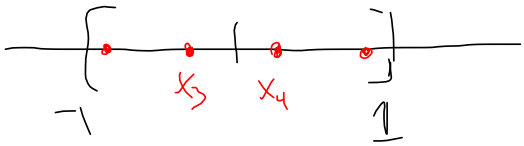
Office

Mon 11-12, 1-3

Tues 10-12

Wed 12-2

Other if you ask



$\{1, x_1, x_2, x_3, \ldots\}$

P_4

$$\langle f, g \rangle = \frac{1}{2} \int_{-1}^{1} f(x) g(x) dx$$

x_1	w_1
x_2	w_2
x_3	w_3
x_4	w_4