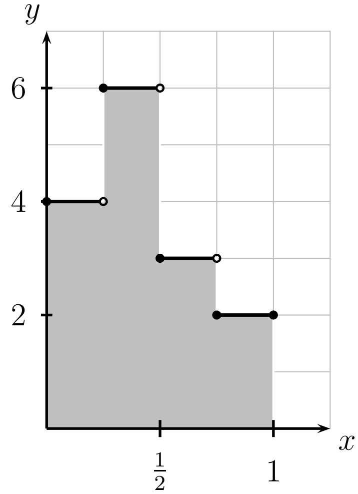
Math 342 Signal Processing Exercises 1 **Spring 2016**

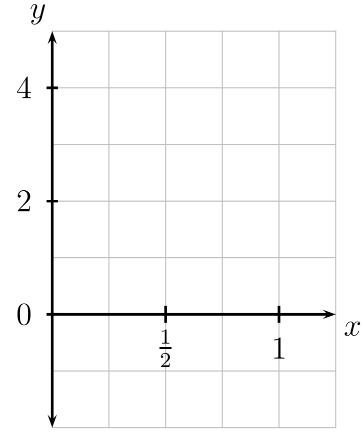
1. The graph of what we will call a **step function** f is shown below and to the left. In the bottom center is the piecewise definition of another step function g. Provide the stepwise definition for f and the graph for g.



Function f

$$f(x) = \left\{ \vphantom{\begin{matrix}a\\a\\a\\a\\a\end{matrix}} \right.$$

$$g(x) = \begin{cases} 3 & \text{for} \quad x \in [0, \frac{1}{4}) \\ 4 & \text{for} \quad x \in [\frac{1}{4}, \frac{1}{2}) \\ -1 & \text{for} \quad x \in [\frac{1}{2}, \frac{3}{4}) \\ 2 & \text{for} \quad x \in [\frac{3}{4}, 1] \end{cases}$$



Function g

2. Let $f(x) = \begin{cases} -2 & \text{for} \quad x \in [0, \frac{1}{4}) \\ 5 & \text{for} \quad x \in [\frac{1}{4}, \frac{1}{2}) \\ 3 & \text{for} \quad x \in [\frac{1}{2}, \frac{3}{4}) \\ 1 & \text{for} \quad x \in [\frac{3}{4}, 1] \end{cases}$ and $g(x) = \begin{cases} 3 & \text{for} \quad x \in [0, \frac{1}{4}) \\ 4 & \text{for} \quad x \in [\frac{1}{4}, \frac{1}{2}) \\ -1 & \text{for} \quad x \in [\frac{1}{2}, \frac{3}{4}) \\ 2 & \text{for} \quad x \in [\frac{3}{4}, 1] \end{cases}$.

   (a) Give the piecewise equation for the function $(f+g)(x)$ in the space provided below and to the left.

$$(f+g)(x) = \left\{ \vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}} \right. \qquad\qquad (fg)(x) = \left\{ \vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}} \right.$$

   (b) Give the piecewise equation for $(fg)(x) = f(x)g(x)$ in the space provided above and to the right.

   (c) Compute $\int_0^1 f(x)\,dx$, **showing clearly the arithmetic used to get its value**.

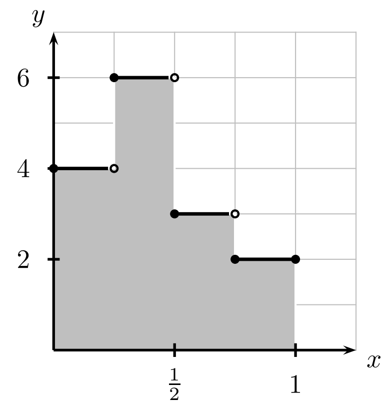
$$\int_0^1 f(x)\,dx =$$

   (d) Using the standard integral inner product, compute $\langle f, g \rangle$ in the space below.

From this point on you will need to refer to the three bases for the step functions that are given on another sheet.

3. Let $g(x) = 4h_0(x) - h_1(x) - 2h_{20}(x) + 3h_{21}(x)$. Give the piecewise definition of g to the left below. (**Hint:** It is probably most efficient to think of each interval of length $\frac{1}{4}$ at a time, and consider the value of each function in $\mathcal{H}$ on that interval.)

$$g(x) = \begin{cases} \\ \end{cases}$$



Function f

4. The function f from Exercise 1 is shown again above and to the right. Find scalars c_1, c_2, c_3 and c_4 such that $f(x) = c_1h_0(x) + c_2h_1(x) + c_3h_{20}(x) + c_4h_{21}(x)$. (**Hint:** Find the value of $c_1h_0(x) + c_2h_1(x) + c_3h_{21}(x) + c_4h_{22}(x)$ on each of the intervals $[0, \frac{1}{4})$... and set it equal to the value of f on that interval. That will give you four equations in four unknowns. Solve!)

5. Give the coordinate vector $[f]_{\mathcal{H}}$ of the function f with respect to the basis $\mathcal{H}$. (You found its components in the previous exercise!)

6. Give the coordinate vector $[f]_{\mathcal{T}}$ of the function f with respect to the basis $\mathcal{T}$.

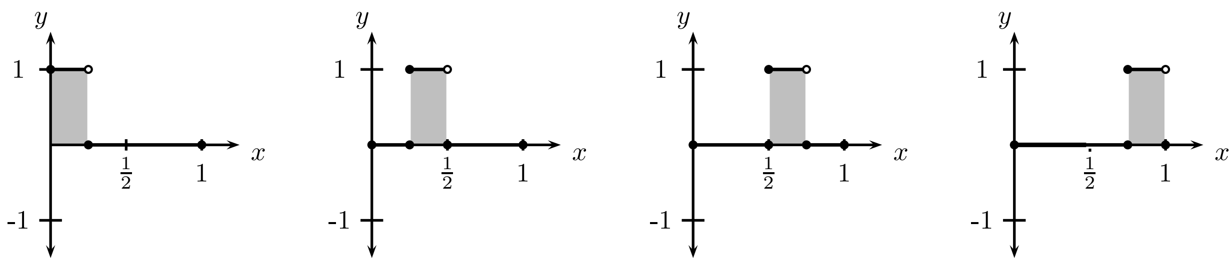
7. Give the coordinate vector $[f]_{\mathcal{E}}$ of the function f with respect to the basis $\mathcal{E}$.

8. As you should have just seen, it is very easy to find the coordinate vector of a step function with respect to the basis $\mathcal{E}$, but finding the coordinate vectors with respect to the other two bases was significantly more difficult. For that reason, it would be nice to determine the two change of basis matrices $[P]_{\mathcal{E},bsst}$ and $[P]_{\mathcal{E},bssh}$. Do that, remembering that it might be easier to find the change of basis matrices for the other direction and then take their inverses!

9. Test your change of basis matrices by applying them to the coordinate vector $[f]_{\mathcal{E}}$ and see if the results match your answers to Exercises 5 and 6.

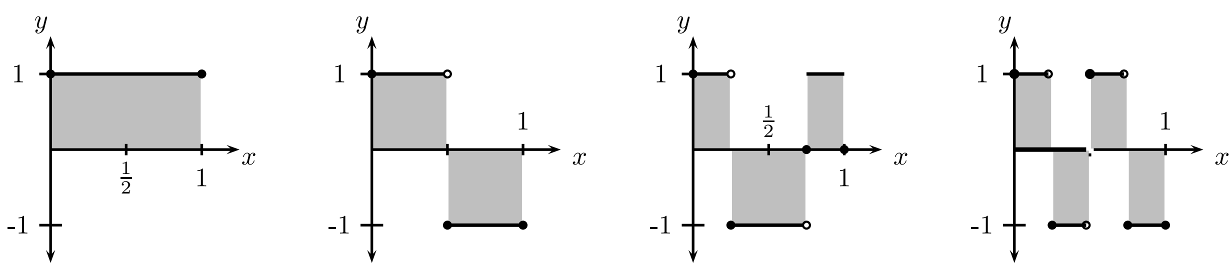
We will be interested in three different bases for our step functions, denoted by $\mathcal{E}$, $\mathcal{T}$ and $\mathcal{H}$. The graphs of the basis elements for each basis are shown below. You should look carefully at the geometry and convince yourself that all three are orthogonal bases.

The basis $\mathcal{E}$:



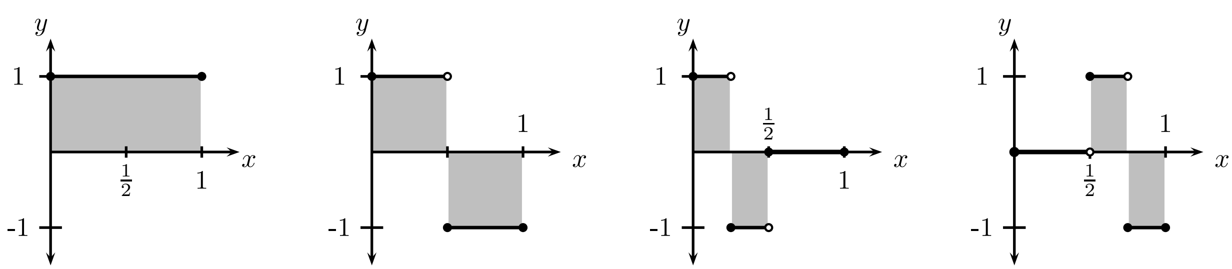
$e_1(x)$ $e_2(x)$ $e_3(x)$ $e_4(x)$

The basis $\mathcal{T}$:



$c_0(x)$ $s_1(x)$ $c_1(x)$ $s_2(x)$

The basis $\mathcal{H}$:



$h_0(x)$ $h_1(x)$ $h_{20}(x)$ $h_{21}(x)$