Math 371 ASSIGNMENT 5, WINTER 2016 **Name:** 12 points

1. The shaded area below is the **feasible region** for Exercise 2 of Assignment 3. We will consider the region, along with the **objective function** $z = 140x + 150y$.

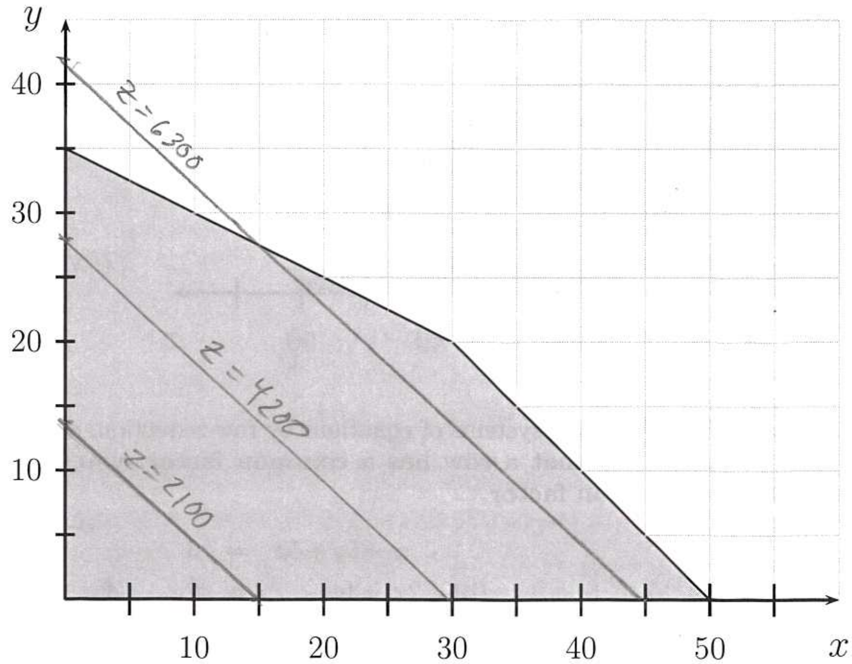
   (a) Substitute the value $z = 2100$ into the objective function and plot the resulting line by finding its intercepts *using a straightedge to draw the line.* Then write $z = 2100$ along the line, which is called a **level curve** of the objective function.

   (b) Draw and label the level curves for $z = 4200$ and $z = 6300$. (They are both lines as well.)

   (c) How do the three lines you've drawn relate to each other?

+1

+1



$140x + 150y = 2100$

x	y
0	14
15	0

$140x + 150y = 4200$

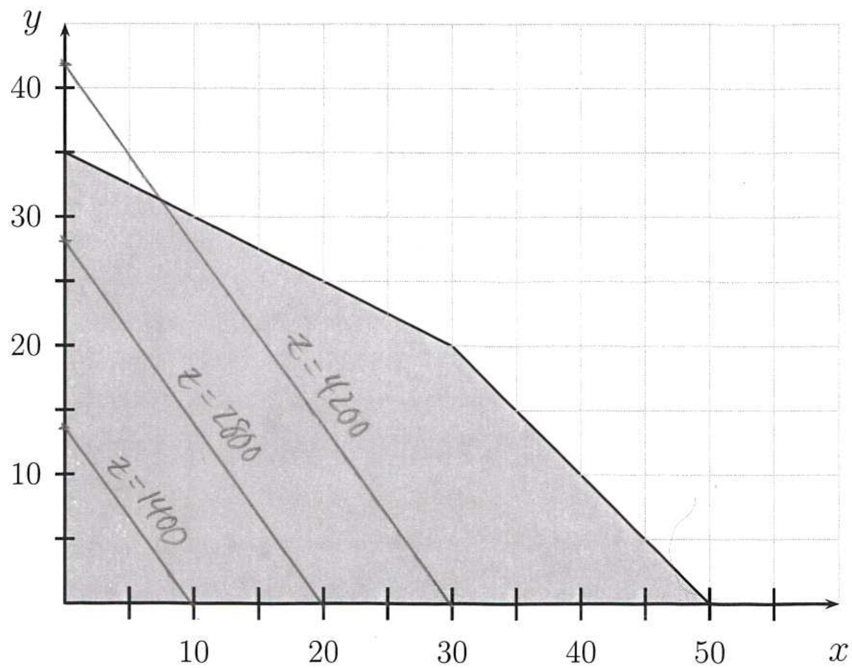
x	y
0	28
30	0

$140x + 150y = 6300$

x	y
0	42
45	0

2. Now consider the objective function $z = 140x + 100y$. Draw in the level curve lines for $z = 1400$, $z = 2800$ and $z = 4200$ on the grid below.

+1



$140x + 100y = 1400$

x	y
0	14
10	0

$140x + 100y = 2800$

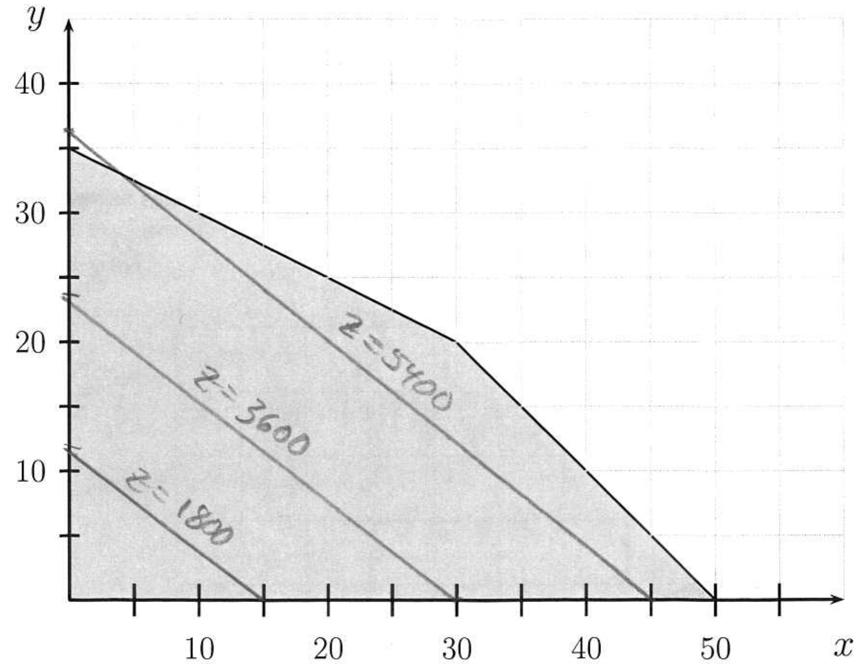
x	y
0	28
20	0

$140x + 100y = 4200$

x	y
0	42
30	0

3. Finally, for the objective function $z = 120x + 150y$. Draw in the level curve lines for $z = 1800$, $z = 3600$ and $z = 5400$ on the grid below.

+1



$120x + 150y = 1800$

x	y
0	12
15	0

$120x + 150y = 3600$

x	y
0	24
30	0

$120x + 150y = 5400$

x	y
0	36
45	0

4. On another sheet of paper, solve each of the following systems of equations by row reduction, as we did in class on Thursday. **Show all steps clearly. Any time that a row has a common factor, you will make things easier if you divide that row by the common factor.**

(a)
$$\begin{aligned} x - 3y + 2z &= 9 \\ -5x + y + 4z &= -17 \\ 2x + 2y - 3z &= 3 \end{aligned}$$
+½

(b)
$$\begin{aligned} x - 5y + 5z &= 3 \\ 2x - 3y + z &= 2 \\ 3x + y - 3z &= 5 \end{aligned}$$
+½

5. Determine where the lines $x + y = 100$ and $5x + 3y = 300$ cross by finding the x and y values that make both equations true. Do this by

(a) the substitution method

$x = 100 - y$

$5(100 - y) + 3y = 300$

$500 - 5y + 3y = 300$

$-2y = -200$

$\boxed{y = 100 \quad x = 0}$

+1

(b) the addition method

$x + y = 100 \implies -5x - 5y = -500$

$5x + 3y = 300 \implies 5x + 3y = 300$

$-2y = -200$

$\boxed{y = 100 \quad x = 0}$

+1

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(4) a)

$$\left[\begin{array}{cccc} 1 & -3 & 2 & 9 \\ -5 & 1 & 4 & -17 \\ 2 & 2 & -3 & 3 \end{array}\right] \xrightarrow[-2R_1+R_3 \to R_3]{5R_1+R_2 \to R_2} \left[\begin{array}{cccc} 1 & -3 & 2 & 9 \\ 0 & -14 & 14 & 28 \\ 0 & 8 & -7 & -15 \end{array}\right] \xrightarrow{R_2 \div 14} \left[\begin{array}{cccc} 1 & -3 & 2 & 9 \\ 0 & -1 & 1 & 2 \\ 0 & 8 & -7 & -15 \end{array}\right]$$

$$\xrightarrow{8R_2+R_3 \to R_3} \left[\begin{array}{cccc} 1 & -3 & 2 & 9 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$\boxed{z=1} \longrightarrow -y+1=2$

$\boxed{y=-1}$

$x+3+2=9$

$\boxed{x=4}$

(4) b)

$$\left[\begin{array}{cccc} 1 & -5 & 5 & 3 \\ 2 & -3 & 1 & 2 \\ 3 & 1 & -3 & 5 \end{array}\right] \xrightarrow[-3R_1+R_3 \to R_3]{-2R_1+R_2 \to R_2} \left[\begin{array}{cccc} 1 & -5 & 5 & 3 \\ 0 & 7 & -9 & -4 \\ 0 & 16 & -18 & -4 \end{array}\right] \xrightarrow{R_3 \div 2} \left[\begin{array}{cccc} 1 & -5 & 5 & 3 \\ 0 & 7 & -9 & -4 \\ 0 & 8 & -9 & -2 \end{array}\right]$$

$$\xrightarrow{8R_2+(-7)R_3 \to R_3} \left[\begin{array}{cccc} 1 & -5 & 5 & 3 \\ 0 & 7 & -9 & -4 \\ 0 & 0 & -9 & -18 \end{array}\right]$$

$-9z=-18$

$\boxed{z=2}$

$7y-18=-4$

$7y=14$

$\boxed{y=2}$

$x-10+10=3$

$\boxed{x=3}$

Section 3.2: 4a, b; 6a, c

(4) a)

point	$z=0.35x+1.25y$
(0,15)	18.75
(6,18)	24.6
(10,9)	14.75 +1
(12,0)	4.2
(0,0)	0

The maximum value is 24.6 at (6,18) and the minimum value is 0 at (0,0)

point	$z=1.5x+0.5y$
(0,0)	0
(0,15)	7.5
(6,18)	18 +1
(10,9)	19.5
(12,0)	18

The maximum value is 19.5 at (10,9) and the minimum value is 0 at (0,0).

6 a)

point	$z = 4x + y$	
(0,10)	10	
(2,4)	12	
(5,2)	22	+1
(15,0)	60	

The minimum value is 10 at (0,10). There is no maximum, because the region is unbounded.

c)

point	$z = x + 2y$	
(0,10)	20	
(2,4)	10	
(5,2)	9	+1
(15,0)	15	

The minimum value is 9 at (5,2). There is no maximum value, because the region is unbounded.