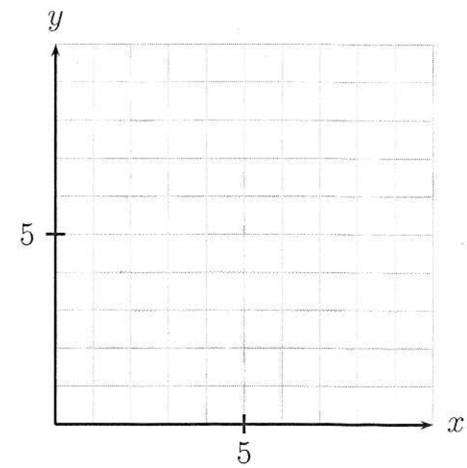
Math 371 ASSIGNMENT 9, WINTER 2016

For this assignment you will be maximizing three different objective functions for the same feasible region, bounded by the inequalities

$$5x + 2y \le 40 \qquad x + 2y \le 16$$

$$x \ge 0 \qquad y \ge 0$$




1. Graph the first two inequalities using Desmos, and sketch the graph of the feasible region on the grid above and to the right. (Draw in the boundaries and shade the region.

**For the rest of the assignment, fill in blanks on this page, show all your work with tableaus on additional paper.**

2. For this exercise the objective function is $z = 3x + 4y$.

   (a) Give the simplex tableau for the constraints and objective function on your additional paper. Then give the solution for the tableau as it is:

   $x =$ ________ $y =$ ________ $z =$ ________

   (b) If $x$ increases by one unit, $z$ increases by ______ units. If $y$ increases by one unit, then $z$ increases by ______ units. The greatest increase in $z$ is then achieved by going from the point you got in (a) along an edge of the feasible region to the point ( ______ , ______ ).

   (c) Determine the first pivot and pivot around it. Give the resulting tableau and its solution.

   $x =$ ________ $y =$ ________ $z =$ ________

   (d) Pivot again and give the final tableau and solution.

   $x =$ ________ $y =$ ________ $z =$ ________

   (e) Here's what the simplex method does: It starts us at $(0, 0)$, where $z$ has the value zero. It then moves us along one of the edges of the feasible region to either $(8, 0)$ or $(0, 8)$, *whichever gives a greater value of* $z$. Add the equation $3x + 4y = z$ in Desmos, with a slider for $z$ that goes from 0 and 50. Change the $z$ line to black. Use it to draw in the line for $z = 12$ on the above graph of the feasible region.

   (f) Now increase the $z$ slider and use it to fill in the blanks below:

   - At the point $(8, 0)$ the value of the objective function is $z =$ ________
   - At the point $(0, 8)$ the value of the objective function is $z =$ ________

   The greater increase in $z$ is obtained by going to which of the two points above?

   Do you see what's going on here?

3. For this exercise, we change the objective function to $z = 4x + 3y$.

(a) Make the above change in Desmos, noting the change in the objective function line.

(b) Give the simplex tableau for the constraints and objective function on your additional paper. Then give the solution for the tableau as it is:

$x =$ ________ $y =$ ________ $z =$ ________

(c) If $x$ increases by one unit, $z$ increases by ______ units. If $y$ increases by one unit, then $z$ increases by ______ units. The greatest increase in $z$ is then achieved by going from the point you got in (a) along an edge of the feasible region to the point ( ______ , ______ ).

(d) Determine the first pivot and pivot around it. Give the resulting tableau and its solution.

$x =$ ________ $y =$ ________ $z =$ ________

(e) Pivot again and give the final tableau and solution.

$x =$ ________ $y =$ ________ $z =$ ________

(f) Use Desmos to draw in the line for $z = 12$ on the graph of the feasible region, *as a dotted line.*

(g) Now increase the $z$ slider and use it to fill in the blanks below:

- At the point $(8, 0)$ the value of the objective function is $z =$ ________
- At the point $(0, 8)$ the value of the objective function is $z =$ ________

The greater increase in $z$ is obtained by going to which of the two points above?

4. Finally, change the objective function to $z = x + 3y$.

(a) Change the objective function in Desmos and use the slider to determine the solution from the graph. Give it below:

$x =$ ________ $y =$ ________ $z =$ ________

(b) Set up the simplex tableau for this objective function and solve it.