Math 371 DERIVATIVES OF POLYNOMIAL FUNCTIONS **Winter 2016**

Associated with every function is another function called its derivative. We will spend some time learning how to find the derivatives of various kinds of functions. The easiest, and perhaps most important, kind of function to be able to find the derivative of is a polynomial function. A polynomial function is a function that is just various powers of the independent variable (usually x), each multiplied by a number (which could be zero or one) and added together. Here are some examples:

$$f(x) = x^4 - 2x^3 + 7x^2 + 5x - 4 \qquad y = x^5 + 7x^3 - 2x \qquad g(x) = x^2 - 3x + 2 \qquad y = \frac{2}{3}x - 4$$

The **degree** of a polynomial is the highest power in the polynomial, so the above polynomials have degrees four, five, two and one. Now consider just the first function f. It is made up of the "parts" x^4, $-2x^3$, $7x^2$, $5x$ and -4. These are called **terms** of the polynomial. The last one in particular is called the **constant term**. The number multiplying each power of x is called the **coefficient** of that power of x. The coefficients of the polynomial function f are 1, -2, 7, 5 and -4.

It is useful to remember that

$$x^0 = 1 \qquad \text{and} \qquad x^1 = x$$

Using this idea and the fact that one times anything is that thing, we can rewrite the first polynomial above as

$$f(x) = 1x^4 - 2x^3 + 7x^2 + 5x^1 - 4x^0$$

Now the derivative of this function is another function that we denote by $f'(x)$. We find it as follows: For each term,

- multiply the power of x times its coefficient, and
- decrease the power of x by one.

That's it! So the derivative of f is

$$f'(x) = 4 \cdot 1x^{4-1} - 3 \cdot 2x^{3-1} + 2 \cdot 7x^{2-1} + 1 \cdot 5x^{1-1} - 0 \cdot 4x^{0-1} = 4x^3 - 6x^2 + 14x + 5$$

Note that the derivative of the constant term is zero, and the derivative of the x term is just its coefficient.

When our function is denoted with y rather than $f(x)$ we usually denote the derivative by $\frac{dy}{dx}$. (You will see later why this is, and it is a very suggestive notation once we know what it means.) So the derivative of $y = x^5 + 7x^3 - 2x$ is

$$\frac{dy}{dx} = 5x^4 + 21x^2 - 2.$$

Got it? Try Examples 1 through 5 here.