Math 411 ASSIGNMENT 10 **Due at 3 PM Wednesday, January 29th**

Show clearly how all answers are obtained. When computing integrals, show all steps clearly.

For each of Exercises 1, 2, and 3, give

(a) the principal root, in both exponential and rectangular form,

(b) the n distinct roots in rectangular form,

(c) all roots, in exponential form.

1. $(-16+16i)^{1/4}$ 2. $(-5i)^{1/3}$ 3. $\sqrt{i}$

4. The nth roots of a complex number z all lie on a circle centered at the origin. In what situations is the radius of the circle greater than $|z|$?

5. Integrate $f(z) = \dfrac{1}{z}$ around the circle of radius $R > 0$ centered at the origin. Show your parametrization, and compute the integral by hand. (It should be a very easy computation.)

6. Let $C_r(z_0)$ be the circle of radius r centered at z_0.

   (a) Give a parametrization γ for $C_r(z_0)$, and give its derivative.

   (b) Compute $\displaystyle\int_{C_r(z_0)} (z-z_0)^n\, dz$ for $n \neq -1$ by hand.

   (c) Compute $\displaystyle\int_{C_r(z_0)} (z-z_0)^n\, dz$ for $n = -1$ by hand.

7. Let $f(x) = \dfrac{5}{3+x}$.

   (a) Give the power series representation of the function f.

   (b) Determine the radius of convergence of the series.

8. Consider the series $4 + \frac{4}{5}x + \frac{4}{25}x^2 + \frac{4}{125}x^3 + \cdots$

   (a) Give the function that the series represents, where it converges.

   (b) Give the radius of convergence.