Math 411 ASSIGNMENT 11 **Due at 3 PM Monday, February 3rd**

Show clearly how all answers are obtained. When computing integrals, show all steps clearly.

For each of Exercises 1, 2, and 3, give

(a) the principal root, in both exponential and rectangular form,

(b) the n distinct roots in rectangular form,

(c) all roots, in exponential form.

1. $(-16)^{1/4}$
2. $(-5i)^{1/3}$
3. $\sqrt{\frac{1}{4}i}$
4. The nth roots of a complex number z all lie on a circle centered at the origin. In what situations is the radius of the circle greater than $|z|$?

Again we will denote by $C_r(z_0)$ be the circle of radius r centered at z_0. You should know by now that such a circle is parameterized by

$$\gamma(\theta) = z_0 + re^{i\theta}, \quad 0 \leq \theta \leq 2\pi.$$

(Some of you did, or attempted, something far more complicated on your exam - make note of this method!)

5. Set up and simplify the integral $\displaystyle\frac{1}{2\pi i}\int_{C_r(z_0)} \frac{f(z)}{z - z_0}\,dz$. Remember that i is just a constant, so it can go in and out of the integral as you please, and your answer will contain the unknown function f.
6. Your answer to Exercise 5 should have been $\displaystyle\frac{1}{2\pi}\int_0^{2\pi} f(z_0 + re^{i\theta})\,d\theta$. Evaluate this integral when $z_0 = 1 + 2i$, $r = 3$ and $f(z) = z^2$, **using Wolfram**. If you would like an algebraic challenge, try it by hand as well.
7. Still letting $z_0 = 1 + 2i$ and $f(z) = z^2$, find $f(z_0)$.
8. Repeat Exercise 6 but for $g(z) = |z|^2$. (Recall again that $|z|^2 = z\overline{z}$.)
9. Repeat Exercise 7 but for $g(z) = |z|^2$.