

6 points

$$\textcircled{1} (-16)^{\frac{1}{4}} = (16e^{i(\pi+2\pi n)})^{\frac{1}{4}} = \boxed{2e^{i(\frac{\pi}{4}+\frac{\pi}{2}n)}} \quad (c)$$

a) $2e^{i\frac{\pi}{4}}$

+1 b) $\sqrt{2} + \sqrt{2}i, -\sqrt{2} + \sqrt{2}i, -\sqrt{2} - \sqrt{2}i, \sqrt{2} - \sqrt{2}i$

$$\textcircled{2} (-5i)^{\frac{1}{3}} = [5e^{i(\frac{3\pi}{2}+2\pi n)}]^{\frac{1}{3}} = \boxed{\sqrt[3]{5} e^{i(\frac{\pi}{2}+\frac{2\pi}{3}n)}} \quad (c)$$

a) $\sqrt[3]{5} e^{i\frac{\pi}{2}} = \sqrt[3]{5} i$

+1 b) $\sqrt[3]{5} i, \sqrt[3]{5}(-\frac{\sqrt{3}}{2} - \frac{1}{2}i), \sqrt[3]{5}(\frac{\sqrt{3}}{2} - \frac{1}{2}i)$

$$\left(-\sqrt[3]{5} e^{i\frac{\pi}{2}}\right)^3 = -5e^{i\frac{3\pi}{2}} = 5i$$

$$\textcircled{3} \sqrt{\frac{1}{4}i} = \left(\frac{1}{4}e^{i(\frac{\pi}{2}+2\pi n)}\right)^{\frac{1}{2}} = \boxed{\frac{1}{2}e^{i(\frac{\pi}{4}+\pi n)}} \quad (c)$$

a) $\frac{1}{2}e^{i\frac{\pi}{4}} = \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4}i$

b) $\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4}i, -\frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4}i$

④ $|z| < 1$

$$\textcircled{5} \frac{1}{2\pi i} \int_{C_r(z_0)} \frac{f(z)}{z-z_0} dz = \frac{1}{2\pi i} \int_0^{2\pi} \frac{f(z_0+re^{i\theta})}{(z_0+re^{i\theta})-z_0} rie^{i\theta} d\theta$$

$$= \frac{1}{2\pi i} \int_0^{2\pi} \frac{f(z_0+re^{i\theta})}{re^{i\theta}} rie^{i\theta} d\theta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} f(z_0+re^{i\theta}) d\theta$$

+1

$$x_{\frac{1}{2}} \textcircled{6} \quad \frac{1}{2\pi} \int_0^{2\pi} (1+2i+3e^{i\theta})^2 d\theta = -3+4i$$

$$x_{\frac{1}{2}} \textcircled{7} \quad f(1+2i) = (1+2i)^2 = (1+2i)(1+2i) = 1+2i+2i-4 = -3+4i$$

$$x_{\frac{1}{2}} \textcircled{8} \quad \frac{1}{2\pi} \int_0^{2\pi} (1+2i+3e^{i\theta})(1-2i+3e^{-i\theta}) d\theta = 14$$

$$x_{\frac{1}{2}} \textcircled{9} \quad g(1+2i) = (1+2i)(1-2i) = 5$$