**Math 411** ASSIGNMENT 12 **Due at 3 PM Wednesday, February 5th**

**Give all answers in exact form**. Recall that we use the notation $C_r(z_0)$ for the circle of radius $r$ centered at $z_0$ and traveled in the counterclockwise direction.

1. Use Cauchy's Integral Formula and the definition of the sine of a complex number to evaluate $\displaystyle\int_{C_3(0)} \frac{\sin z}{z-i}\,dz$.

2. In this exercise you will consider the integral $\displaystyle\int_{C_r(0)} \frac{z+1}{z^2-2z-8}\,dz$.

   (a) Noting that $\displaystyle\int_{C_r(0)} \frac{z+1}{z^2-2z-8}\,dz = \int_{C_r(0)} \frac{z+1}{(z-4)(z+2)}\,dz$, we can see (?) that the integral is zero when $r=1$. Give a result from your definitions and theorems sheets that tells us this.

   (b) Do a partial fractions decomposition of the integrand and find the integral when $r=3$ by splitting into two separate integrals.

   (c) Again using your partial fraction decomposition, do the integral when $r=5$.

   (d) Use the hint given in Exercise 30 on page 57 of 4G, to calculate the integral with $r=3$ again. (This is the method of Example 2 in the suggested reading under 2/3 in the schedule.)

2. Evaluate $\displaystyle\int_{C_2(0)} \frac{e^z}{z(z-3)}\,dz$ using the method of Example 2 from the suggested reading again.

3. You *will not* be using Cauchy's integral formula for this exercise - you'll actually have to compute (with the help of Wolfram) the integral.

   (a) Parameterize $C_2(3+i)$ as a curve $\gamma(\theta)$ and then calculate $\displaystyle\frac{1}{2\pi}\int_0^{2\pi} [\gamma(\theta)]^2\,d\theta$ using Wolfram.

   (b) Calculate $f(3+i)$ where $f(z)=z^2$.

   (c) Your answers to (a) and (b) should be the same. What result from the definition and theorem sheets does this illustrate?

4. For this exercise you will be considering the rational function $\displaystyle\frac{1}{z-i}$.

   (a) Noting that $\displaystyle\frac{1}{z-i} = \frac{1}{-i+z}$, perform long division of 1 by $-i+z$, continuing until you get the fifth power term.

   (b) Multiply the numerator and denominator of $\displaystyle\frac{1}{-i+z}$ by $i$, then write the result in the form $a\left(\dfrac{1}{1-r}\right)$. Then use $\displaystyle\frac{1}{1-r} = 1+r+r^2+r^3+r^4+\cdots$ to get the first six terms of a series. Then multiply $a$ in and simplify terms of the form $(\pm iz)^n$ so that there is only a power on the $z$.

   (c) Your results from (a) and (b) should be the same - if they aren't, find and correct any errors.