Math 411, Assignment 12 — 7 points $-\frac{1}{2}$ for "free" — W14

(1) $\int_{C_3(0)} \frac{\sin z}{z-i}\,dz = 2\pi i \sin(i) = 2\pi i \cdot \frac{1}{2i}\left(\exp(i^2) - \exp(-i^2)\right) = \pi\left(e^{-1} - e\right) = \pi\left(\frac{1}{e} - e\right)$ $+\frac{1}{2}$

(2) a) Corollary 4.10. $+\frac{1}{2}$

b) $\frac{z+1}{(z-4)(z+2)} = \frac{A}{z-4} + \frac{B}{z+2} \Rightarrow z+1 = A(z+2) + B(z-4)$ $+1\frac{1}{2}$

Let $z=-2 \Rightarrow -1 = -6B \Rightarrow B = \frac{1}{6}$

Let $z=4 \Rightarrow 5 = 6A \Rightarrow A = \frac{5}{6}$

$+\frac{1}{2}$ $$\int_{C_3(0)} \frac{z+1}{z^2-2z-8}\,dz = \frac{5}{6}\int_{C_3(0)} \frac{1}{z-4}\,dz + \frac{1}{6}\int_{C_3(0)} \frac{1}{z+2}\,dz$$

$+2\frac{1}{2}$ total $$= 0 + 2\pi i \cdot \frac{1}{6} = \frac{\pi i}{3}$$

c) $\int_{C_5(0)} \frac{z+1}{z^2-2z-8}\,dz = \frac{5}{6}\int_{C_5(0)} \frac{1}{z-4}\,dz + \frac{1}{6}\int_{C_5(0)} \frac{1}{z+2}\,dz = \frac{5}{6}\cdot 2\pi i + \frac{1}{6}\cdot 2\pi i = 2\pi i$ $+\frac{1}{2}$

d) $\int_{C_3(0)} \frac{z+1}{z^2-2z-8}\,dz = \int_{C_3(0)} \frac{(z+1)/(z-4)}{z+2}\,dz = 2\pi i \cdot \frac{-2+1}{-2-4} = \frac{\pi i}{3}$ $+\frac{1}{2}$

(3) $\int_{C_2(0)} \frac{e^z}{z(z-3)}\,dz = \int_{C_2(0)} \frac{e^z/(z-3)}{z}\,dz = 2\pi i \cdot \frac{e^0}{0-3} = -\frac{2\pi i}{3}$ $+\frac{1}{2}$

(4) a) $\gamma(\theta) = 3+i+2e^{i\theta}$, $0 \le \theta \le 2\pi$ $+\frac{1}{2}$

$\frac{1}{2\pi}\int_0^{2\pi} f(\gamma(\theta))\,d\theta = \frac{1}{2\pi}\int_0^{2\pi} (3+i+2e^{i\theta})^2\,d\theta = 8+6i$ $+\frac{1}{2}$

$+2$ total

b) $f(3+i) = (3+i)^2 = (3+i)(3+i) = 9+6i-1 = 8+6i$ $+\frac{1}{2}$

c) Corollary 4.14 $+\frac{1}{2}$

⑤ a)

$$-i+z \;\overline{)\; 1 \quad i + z - iz^2 - z^3 + iz^4 + z^5 - \cdots}$$

$$\begin{array}{l} 1 + iz \\ \underline{-iz} \\ -iz + z^2 \\ \underline{-z^2} \\ -z^2 - iz^3 \\ \underline{iz^3} \\ iz^3 - z^4 \\ \underline{z^4} \\ z^4 + iz^5 \\ \underline{-iz^5} \end{array}$$

x½

b)

$$\frac{1}{-i+z} = \frac{i}{i} \cdot \frac{1}{-i+z}$$

$$= \frac{i}{1+iz}$$

x½

$$= i\left(\frac{1}{1-(-iz)}\right)$$

$$= i\left(1 - iz + (-iz)^2 + (-iz)^3 + (-iz)^4 + (-iz)^5 + \cdots\right)$$

$$= i\left(1 - iz - z^2 + iz^3 + z^4 - iz^5 + \cdots\right)$$

$$= i + z - iz^2 - z^3 + iz^4 + z^5 - \cdots$$

$i = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$
$i^5 = i$