Math 411 ASSIGNMENT 13 **Due at 3 PM Friday, February 7th**

Give all answers in exact form. Recall that we use the notation $C_r(z_0)$ for the circle of radius r centered at z_0 and traveled in the counterclockwise direction.

1. Calculate $\displaystyle\int_{C_2(0)} \frac{ze^z}{(2z-3)}\,dz$. Begin by factoring 2 out of the denominator and putting it into the numerator.

2. Use Cauchy's Integral Formula for the second derivative to calculate $\displaystyle\int_{C_2(0)} \frac{z^3+5z+1}{(z-i)^3}\,dz$.

3. Consider the integral $\displaystyle\int_{C_3(0)} \frac{\sin z}{z^2(z-2i)}\,dz$.

   (a) Sketch a graph of the contour around which we are integrating and indicate the locations of any "bad spots" of the integrand. Sketch in two new contours γ_1 and γ_2 such that the sum of the integrals around them is equal to the integral around the circle.

   (b) Find the integral around each of the new contours γ_1 and γ_2. For one of them you will need to use the Cauchy Integral Formula for the derivative, and you'll need the quotient rule.

   (c) Give the value of $\displaystyle\int_{C_3(0)} \frac{\sin z}{z^2(z-2i)}\,dz$.

4. Consider the integral $\displaystyle\int_{C_3(2i)} \frac{\cos z}{z^3+9z}\,dz$.

   (a) Factor the denominator of the integrand - it factors into **three** parts. Don't forget that we are working in the complex numbers! That means that something like x^2+4 can be a difference of squares: $x^2+4 = x^2-(-4)$.

   (b) Sketch a graph of the contour around which we are integrating and indicate the locations of any "bad spots" of the integrand. Sketch in two new contours γ_1 and γ_2 such that the sum of the integrals around them is equal to the integral around the circle.

   (c) Find the integral around each of the new contours γ_1 and γ_2. If you end up with i in a denominator, "rationalize the denominator" by multiplying numerator and denominator both by i.

   (d) Give the value of $\displaystyle\int_{C_3(2i)} \frac{\cos z}{z^3+9z}\,dz$.

5. Look up the definitions of the hyperbolic sine and cosine somewhere. (You can find them on page 72 of Churchill and Brown or page 39 of the other book.)

   (a) Give your answer to 4(d) in terms of the hyperbolic cosine.

   (b) Give your answer to 3(c) in terms of the hyperbolic sine.