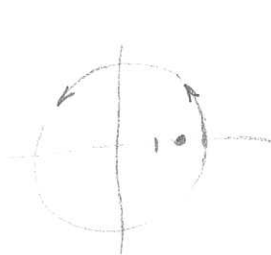


① $\int_{C_2(0)} \frac{ze^z}{z-3} dz = \int_{C_2(0)} \frac{ze^{z/2}}{z-\frac{3}{2}} dz = 2\pi i \cdot \frac{3}{4} e^{\frac{3}{2}} = \boxed{\frac{3\pi}{2} e^{\frac{3}{2}} i}$



$f(z) = \frac{ze^z}{z-\frac{3}{2}}$ $\times \frac{1}{2}$

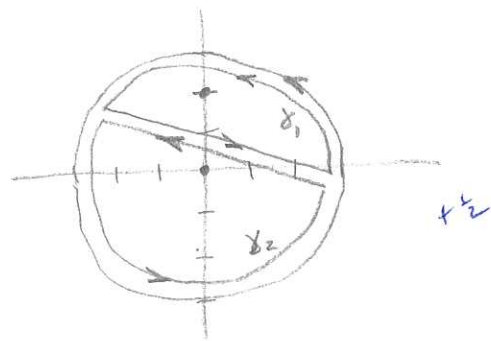
$f(\frac{3}{2}) = \frac{3}{4} e^{\frac{3}{2}}$

② $\int_{C_2(0)} \frac{z^3+5z+1}{(z-i)^3} dz = \pi i \cdot f''(i) = \pi i \cdot 6i = -6\pi$

$f(z) = z^3+5z+1$ $\times \frac{1}{2}$

$f'(z) = 3z^2+5$

$f''(z) = 6z$



③ $\int_{C_3(0)} \frac{\sin z}{z^2(z-2i)} dz$ a) \rightarrow

b) $\int_{\gamma_1} \frac{\sin z}{z^2(z-2i)} dz = \int_{\gamma_1} \frac{\sin z/z^2}{z-2i} dz = 2\pi i \cdot \frac{\sin 2i}{(2i)^2}$
 $= \frac{2\pi i}{-4} \cdot \frac{e^{i(2i)} - e^{-i(2i)}}{2i} = -\frac{\pi}{4}(e^{-2}-e^2)$

c) $\int_{\gamma_2} \frac{\sin z}{z^2(z-2i)} dz = \int_{\gamma_2} \frac{\sin z/(z-2i)}{z^2} dz = 2\pi i \cdot f'(0) = (2\pi i) \cdot (\frac{i}{2}) = -\pi$

$f(z) = \frac{\sin z}{z-2i}$

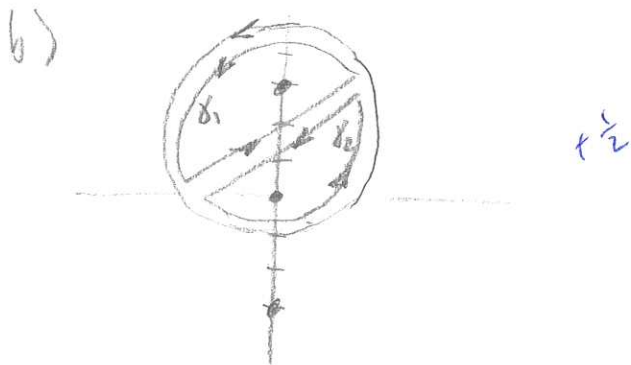
$(z-2i)(z-2i)$
 $z^2-4iz-4$

$f'(z) = \frac{(z-2i)\cos z - \sin z}{z^2-4iz-4}$

$f'(0) = \frac{-2i\cos 0 - \sin 0}{-4} = \frac{i}{2}$

c) $\int_{C_3(0)} \frac{\sin z}{z^2(z-2i)} dz = \boxed{-\pi - \frac{\pi}{4}(e^{-2}-e^2)}$ $\times \frac{1}{2}$

④ a) $\frac{\cos z}{z^3 + 9z} = \frac{\cos z}{z(z+3i)(z-3i)} \quad + \frac{1}{2}$



c) $\int_{\gamma_1} \frac{\cos z}{z^3 + 9z} dz = \int_{\gamma_1} \frac{\cos z / [z(z+3i)]}{z-3i} dz = 2\pi i \cdot \frac{\cos 3i}{3i(3i+3i)}$

$= \frac{2\pi i}{3i(6i)} \cdot \frac{e^{i(3i)} + e^{-i(3i)}}{2} = \frac{\pi}{18i} (e^{-3} + e^3) = -\frac{\pi i}{18} (e^{-3} + e^3)$

\downarrow

$\int_{\gamma_2} \frac{\cos z}{z^3 + 9z} dz = \int_{\gamma_2} \frac{\cos z / [z^2 + 9]}{z} dz = 2\pi i \frac{\cos 0}{0^2 + 9} = \frac{2\pi i}{9}$

\downarrow d) $\int_{C_3(2i)} \frac{\cos z}{z^3 + 9z} dz = \frac{2\pi i}{9} - \frac{\pi i}{18} (e^{-3} + e^3)$

⑤ a) $\int_{C_3(2i)} \frac{\cos z}{z^3 + 9z} dz = \frac{2\pi i}{9} - \frac{\pi i}{9} \cosh(3) \quad + \frac{1}{2} \text{ each}$

b) $\int \frac{\sin z}{z^2(z-2i)} dz = -\pi + \frac{\pi}{2} \sinh(2)$