Math 411 Assignment 14 **Due at 3 PM Monday, February 10th**

For every integral you evaluate, sketch a graph showing the original contour and the locations of all singularities of the integrand. Then sketch in and label any new contours used to evaluate the integral.

1. In this exercise you will evaluate the integral $\displaystyle\int_{C_5(0)} \frac{z+2}{z^2-2z+5}\,dz$.

   (a) We call the numbers that make a polynomial like z^2-2z+5 zero the **roots** of the polynomial. Use the quadratic formula to find the roots of z^2-2z+5. They will be complex numbers!

   (b) If r_1 and r_2 are roots of x^2+bx+c, then x^2+bx+c factors to $(x-r_1)(x-r_2)$. Use this to evaluate the integral, **simplifying your answer to** $a+bi$ **form** (where both a and b are exact).

2. Using what you learned in Exercise 1, evaluate the integral $\displaystyle\int_{C_r(z_0)} \frac{z+2}{z^2-2z+5}\,dz$

   (a) when $r=2$ and $z_0=0$

   (b) when $r=2$ and $z_0=1+i$

3. Evaluate $\displaystyle\int_{C_3(0)} \frac{1}{(z^2+4)^2}\,dz$.

4. Evaluate $\displaystyle\int_{C_2(0)} \frac{\cos z}{z^3+z}\,dz$.

Hint for #4: Give your answer in terms of the cosine of a complex number. Cosine is still an even function for complex numbers, so $\cos(-z)=\cos z$ for all z.

For each of Exercises 5, 6 and 7,

(a) find *all* values of the given root,

(b) give all distinct roots *in rectangular form*, with the principal root first,

(c) show the locations of all roots. They will lie on a circle; make the radius clear and locate the roots with dots on the circle.

5. $(-8)^{\frac{1}{3}}$

6. $(-81i)^{\frac{1}{4}}$

7. $(1)^{\frac{1}{6}}$

8. (a) Add up all of the distinct roots from Exercise 5. Hmmm...

   (b) Add up the distinct roots from Exercise 6.

   (c) Make a conjecture, as a complete sentence.

9. (a) Sketch a unit circle fairly large (like a little larger than a quarter). Put an $\times$ on it somewhere in the second quadrant and label it as $e^{i\theta}$.

   (b) Put a dot at the location of the principal fifth root of $e^{i\theta}$, and label that point with its value as a complex number (in exponential form).

   (c) As accurately as you can, put dots at the other fifth roots of $e^{i\theta}$. What is the angle between any two adjacent (on the unit circle) roots?