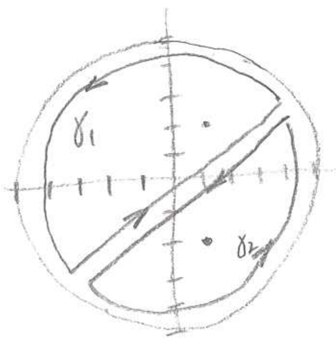
Math 411, Assignment 14 W14

12 points

① a) $z = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(5)}}{2} = \dfrac{2 \pm \sqrt{-16}}{2} = \dfrac{2 \pm 4i}{2} = 1 \pm 2i$ $+\frac{1}{2}$

b) $\displaystyle\int_{C_5(0)} \frac{z+2}{(z-(1+2i))(z-(1-2i))}\,dz = \int_{\gamma_1} \frac{(z+2)/(z-1+2i)}{z-(1+2i)}\,dz + \int_{\gamma_2} \frac{(z+2)/(z-1-2i)}{z-(1-2i)}\,dz$

$+1\frac{1}{2}$



$= 2\pi i \cdot \dfrac{1+2i+2}{1+2i-1+2i} + 2\pi i \cdot \dfrac{1-2i+2}{1-2i-1-2i}$

$= \cancel{2}\pi \cancel{i} \cdot \dfrac{3+2i}{\cancel{4}\cancel{i}} + \cancel{2}\pi \cancel{i} \cdot \dfrac{3-2i}{-\cancel{4}\cancel{i}}$ $\frac{2\pi}{2}i - \frac{3\pi}{2}i$

$= \underbrace{\frac{\pi}{2}(3+2i)} - \frac{\pi}{2}(3-2i)$

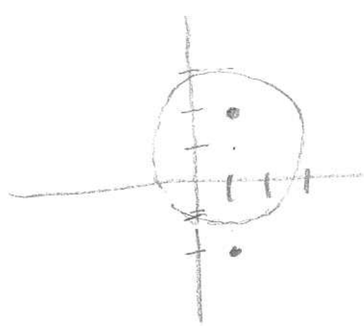
$= 2\pi i$

② a) $\displaystyle\int_{C_2(0)} \frac{z+2}{z^2-2z+5}\,dz = 0$ because $\dfrac{z+2}{z^2-2z+5}$ is holomorphic on $C_2(0)$ and its interior.

$+\frac{1}{2}$

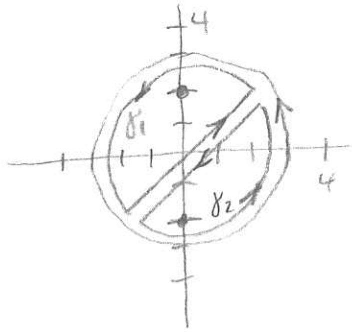
b) $\displaystyle\int_{C_2(1+i)} \frac{z+2}{z^2-2z+5}\,dz = \int_{C_2(1+i)} \frac{(z+2)/(z-1+2i)}{z-(1+2i)}\,dz = \frac{3\pi}{2} + \pi i$

$+\frac{1}{2}$



Math 411, Assignment 14, continued

(3) $$\int_{C_3(0)} \frac{1}{(z^2+4)^2}\,dz = \int_{C_3(0)} \frac{1}{(z+2i)^2(z-2i)^2}\,dz$$



+2

$$= \int_{\gamma_1} \frac{1/(z+2i)^2}{(z-2i)^2}\,dz + \int_{\gamma_2} \frac{1/(z-2i)^2}{(z+2i)^2}\,dz$$

$$= 2\pi i \cdot \frac{1}{32i} + 2\pi i \cdot \left(-\frac{1}{32i}\right) = 0$$

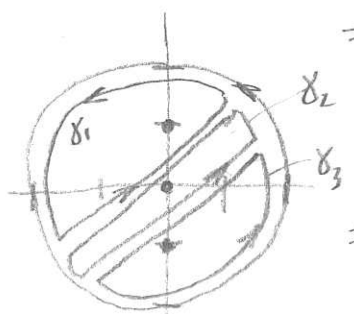
$$f(z) = \frac{1}{(z+2i)^2} = (z+2i)^{-2} \qquad g(z) = \frac{1}{(z-2i)^2} = (z-2i)^{-2}$$

$$f'(z) = -2(z+2i)^{-3} = -\frac{2}{(z+2i)^3} \qquad g'(z) = -2(z-2i)^{-3} = \frac{-2}{(z-2i)^3}$$

$$f'(2i) = -\frac{2}{(2i+2i)^3} = -\frac{2}{(4i)^3} = \frac{1}{32i} \qquad g'(-2i) = \frac{-2}{(-4i)^3} = -\frac{2}{32i}$$

(4) $$\int_{C_2(0)} \frac{\cos z}{z^3+z}\,dz = \int_{C_2(0)} \frac{\cos z}{z(z+i)(z-i)}\,dz$$

x2



$$= \int_{\gamma_1} \frac{\cos z/(z(z+i))}{z-i}\,dz + \int_{\gamma_2} \frac{\cos z/(z^2+1)}{z}\,dz + \int_{\gamma_3} \frac{\cos z/(z(z-i))}{z+i}\,dz$$

$$= 2\pi i \cdot \frac{\cos i}{i(i+i)} + 2\pi i \cdot \frac{\cos 0}{0^2+1} + 2\pi i \cdot \frac{\cos(-i)}{-i(-i-i)}$$

$(-i)(-2i)$

$$= -\pi i \cos i + 2\pi i - \pi i \cos(-i)$$

$$= 2\pi i - 2\pi i \cos i$$

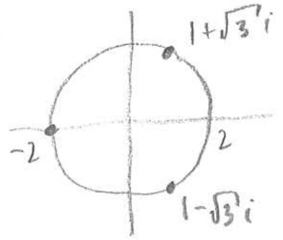
$$= 2\pi i - 2\pi i \cosh(-1)$$

Math 411, Assignment 14, continued

⑤ $(-8)^{\frac{1}{3}} = \left(8e^{i\pi}\right)^{\frac{1}{3}} = 2e^{\left(\frac{\pi}{3}+\frac{2\pi}{3}n\right)i}$ (a)

(b) $2e^{\frac{\pi}{3}i}$, $2e^{\pi i}$, $2e^{\frac{5\pi}{3}i}$ $1+\sqrt{3}i, -2, 1-\sqrt{3}i$

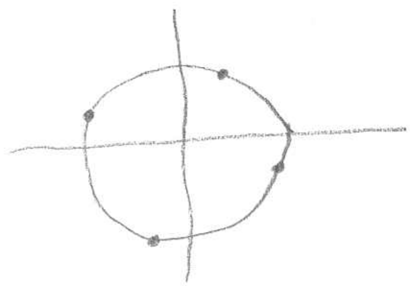
(c)



$+1\frac{1}{2}$

⑥ a) $(-81i)^{\frac{1}{4}} = \left(81e^{\left(\frac{3\pi}{2}+2\pi n\right)i}\right)^{\frac{1}{4}} = 3e^{\left(\frac{3\pi}{8}+\frac{\pi}{2}n\right)i}$

b) ? c)

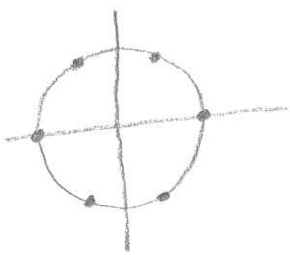


$+1\frac{1}{2}$

⑦ a) $(1)^{\frac{1}{6}} = \left(e^{2\pi n i}\right)^{\frac{1}{6}} = e^{\frac{\pi}{3}ni}$

b) $1, \frac{1}{2}+\frac{\sqrt{3}}{2}i, -\frac{1}{2}+\frac{\sqrt{3}}{2}i, -1, -\frac{1}{2}-\frac{\sqrt{3}}{2}i, \frac{1}{2}-\frac{\sqrt{3}}{2}i$

c)



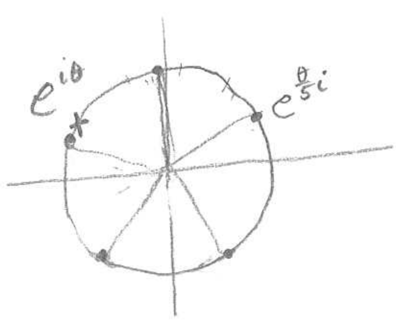
$+1\frac{1}{2}$

⑧ a) $(1+\sqrt{3}i)+(-2)+(1-\sqrt{3}i)=0$

b) Sum of these is zero

c) The sum all the distinct roots is zero

⑨ a)



angle between them is $\frac{2\pi}{5} = 72°$

$+\frac{1}{2}$