Math 411 ASSIGNMENT 15 **Due at 3 PM Wednesday, February 12th**

The triangle inequality is $|z_1 + z_2| \le |z_1| + |z_2|$**.**

1. (a) Apply properties of the modulus, and the triangle inequality, to obtain a bound for $\cos z = \dfrac{\exp(iz) + \exp(-iz)}{2}$.

   (b) Repeat for $\sin z = \dfrac{\exp(iz) - \exp(-iz)}{2i}$.

   (c) Do your results surprise you? Comment on them.

2. For this exercise, $\gamma = \gamma(\theta) = 2i + 3e^{i\theta}$, $0 \le \theta \le 2\pi$.

   (a) Apply the triangle inequality to obtain a bound for $z^2 + 5z - 2$ on γ.

   (b) Using your answer to (a), obtain a bound for $\dfrac{z^2 + 5z - 2}{z - 2i}$ on γ.

   (c) Apply Proposition 4.7(d) to obtain a bound for $\displaystyle\int_\gamma \frac{z^2 + 5z - 2}{z - 2i}\, dz$.

   (d) Use Cauchy's Integral Formula to evaluate $\displaystyle\int_\gamma \frac{z^2 + 5z - 2}{z - 2i}\, dz$.

   (e) Find the modulus of your answer to (d). Is it less than the bound you obtained in (c)?

3. The object of this exercise is to find a value $R_0 > 0$ for which
$$\left|\frac{1}{3z^2 - 5z}\right| \le \frac{1}{100} \qquad (1)$$
   for all points z on the curve $\gamma(\theta) = R_0 e^{i\theta}$, $0 \le \theta \le \pi$.

   (a) In order for (1) to be true, what has to be true about $|3z^2 - 5z|$?

   (b) Apply Lemma 5.6 to obtain the desired value of R_0.

   (c) Find, in terms of ε, the value of R_0 such that $\left|\dfrac{1}{3z^2 - 5z}\right| \le \varepsilon$ for z on $\gamma(\theta) = R_0 e^{i\theta}$, $0 \le \theta \le \pi$.

4. (a) Give three branches of the cube root using set builder notation, with the first branch starting at $\text{Arg}z = 0$ (assuming the argument is increasing within each branch) **and including that value of the argument**.

   (b) Using the third branch you gave in part (a), find $(-8i)^{\frac{1}{3}}$. **Give your answer in rectangular form.**

   (c) Indicate the three branches from (a) graphically, and put a dot at the root you found in (b).

   (d) Give three different branches of the cube root, this time with the first branch starting at $\text{Arg}z = -\pi$ but not including that value of the argument.

   (e) Using the first branch you gave in part(d), find $(-8i)^{\frac{1}{3}}$. **Give your answer in rectangular form.**

   (f) Give the three branches of the cube root, with the first starting at and including $\text{Arg}z = \theta$.

5. Evaluate $\displaystyle\int_{C_3(0)} \frac{z^2 + 1}{z^4 + 4z^2}\, dz$.