Math 411 ASSIGNMENT 18 **Due at 3 PM Wednesday, February 19th**

1. Consider the function $f(z) = \dfrac{3}{5-z}$. **Show clearly how you do each of the following.**

   (a) Use Lemma 7.25 to find the power series representation of this function. Write out the first five terms followed by $\cdots$, then give the series in summation form.

   (b) Find the radius of convergence of the power series.

   (c) Assuming that we can find the derivative of a function by differentiating each term of its power series, find the derivative of f by differentiating the series you found in (a). Again, write out the first five terms of the series followed by $\cdots$, then try to write it in summation form, **starting at** $k = 1$ **for your index.**

   (d) Find the derivative of $f(z)$ as it was given at the beginning of this exercise. Use it to find $f'(1)$.

   (e) Use the first four terms of your your answer to (c) to approximate $f'(1)$. If your answer isn't close to the exact value, then something is wrong somewhere!

   (f) What is z_0 for this series? Find the second, third and fourth derivatives of f from the first part of your answer to (d), using the notation $f^{(3)}$ and $f^{(4)}$ for the third and fourth derivatives. Then use $f^{(0)} = f$, $f^{(1)} = f'$, $f^{(2)} = f''$, $f^{(3)}$ and $f^{(4)}$ to find c_0, c_1, ..., c_4 with the formula given in Corollary 8.5. They should, of course, agree with what you obtained in (a).

2. Consider the function $g(z) = z^2$, which is entire.

   (a) Find a function $G(z)$ such that $G'(z) = g(z)$. There are many such functions (do you see why?), but just give one *specific* one. This function G is called a **primitive** or **antiderivative** of g.

   (b) Consider the curve $\gamma(\theta) = e^{i\theta}$, $0 \le \theta \le \frac{\pi}{2}$. Use the corrected version of Theorem 5.13, which should have $F(\gamma(b)) - F(\gamma(a))$ where it says $F(b) - F(a)$, to compute $\displaystyle\int_\gamma g(z)\,dz$, **showing clearly how you do it**. If you have your things well enough organized to do so, check your answer with what you got for Exercise 3 of Assignment 9, which was the same integral and contour.

3. In class we discussed the fact that the sequence (f_n) of real-valued functions $f_n(x) = x^n$, $0 \le x \le 1$, converges pointwise to the function

$$f(x) = \begin{cases} 0 & \text{if } 0 \le x < 1, \\ 1 & \text{if } x = 1 \end{cases}$$

   (a) Letting $\varepsilon = 0.0001$ and $x = 0.3$, find the minimum value of n for which $|f_n(x) - f(x)| < \varepsilon$.

   (b) Letting $\varepsilon = 0.0001$ and $x = 0.8$, find the minimum value of n for which $|f_n(x) - f(x)| < \varepsilon$. How many times larger is this n than the one you obtained in part (a)?

   (c) Letting $\varepsilon = 0.001$ (note the change), find the minimum value of n for which $|f_n(x) - f(x)| < \varepsilon$ for all x in the interval $[0, 0.95]$.