Math 411 ASSIGNMENT 19 **Due at 3 PM Wednesday, February 26th**

1. The function $f(z) = \dfrac{z+1}{z-1}$ clearly has a singularity at $z = 1$, so we can represent it with a Laurent series centered at one.

   (a) Note that if we write $f(z) = (z+1) \cdot \dfrac{1}{z-1}$, the second factor is already in the correct form for a series centered at one. Manipulate the first factor a little to get a $(z-1)$ term, then multiply that by $\dfrac{1}{z-1}$ to obtain the Laurent series for f.

   (b) Give the values of all of the c_k for your series individually - there aren't very many of them.

   (c) Give the region in which your Laurent series converges.

2. Consider again the function $f(z) = \dfrac{z+1}{z-1}$.

   (a) The function is holomorphic at $z = 0$, so it has a power series representation in some open disc centered there - find it. (**Hint:** Find the power series for $\dfrac{1}{z-1}$ and multiply by $z+1$.)

   (b) Give your power series as a number plus a series from $k = 1$ to infinity **in summation form**.

   (c) Give the region of convergence of your power series.

3. In the previous exercise you found a power series for $f(z) = \dfrac{z+1}{z-1}$ centered at zero. In this exercise you will find a Laurent series for the same function in an annular region outside the region of convergence in which the power series converges. The approach is similar to what you did for Exercise 1, except you will expand the other factor.

   (a) The numerator is already a power series centered at zero. Find the Laurent series centered at zero for $\dfrac{1}{z-1}$ by first factoring z out of the denominator.

   (b) Show how to determine where the Laurent series you found in (a) converges.

   (c) Now multiply your answer to (a) by $z+1$ to get the Laurent series for f. Like you did for Exercise 2, write it as a number plus a Laurent series in summation form.

4. Use Cauchy's Integral Formula to evaluate $\displaystyle\int_{C_2(0)} \frac{z+1}{z-1}\, dz$.

5. Now let $g(z) = \dfrac{1}{z(1+z^2)}$.

   (a) Give two annular regions around $z_0 = 0$ in which we should be able to find a Laurent series representation for g.

   (b) $\dfrac{1}{1+z^2}$ is holomorphic at zero, so it has a power series there. Give the series and its region of convergence.

   (c) Use your answer to (b) to give a Laurent series for g in the smaller of the two annular regions you gave in (a). Give the region in which that series converges.

   (d) Find the Laurent series in the larger of the two annular regions. (**Hint:** You will need to factor z^2 out of the denominator of $\dfrac{1}{1+z^2}$.)

6. Use Cauchy's Integral Formula with our previous method for multiple singularities to find $\displaystyle\int_{C_2(0)} \frac{dz}{z(1+z^2)}$.