Math 411 ASSIGNMENT 22 **Due at 3 PM Thursday, March 6th**

Show your work for each of the following in a neat, organized manner that fully justifies your results/conclusions.

1. Apply Theorem A.6(3) to show that $f(z) = \dfrac{z^3+1}{z^2(z+1)}$ has a removable singularity at -1.

2. Apply Theorem A.7(3) to show that $h(z) = \dfrac{\cos z}{z^2+1}$ has a pole of order one at $-i$.

3. Show that $u(x,y) = \ln(x^2+y^2)$ is harmonic everywhere.

4. Determine the conditions that must hold on the constants a, b, c and d so that $u(x,y) = ax^3+bx^2y+cxy^2+dy^3$ is harmonic everywhere.

5. (**Fun!**) Give two functions f and g that have simple poles at $z=1$ and such that $f+g$ does not have a pole at $z=1$. In this case, what is going on with f and g at $z=1$?

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