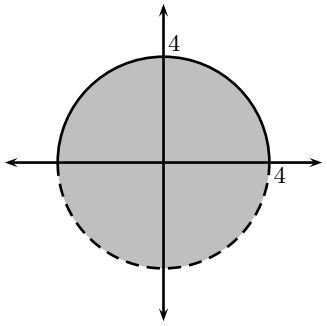
Math 411 ASSIGNMENT 4 **Due at 3 PM Wednesday, January 13th**

1. Determine whether each of the following points is an interior point, exterior point or boundary point of the set S_1 shown to the right.

   (a) $z = 2\sqrt{3} - 2i$ (b) $z = 3i$ (c) $z = -4$ (d) $z = 5e^{\frac{2\pi}{3}i}$

2. Give the interior of S_1, using set builder notation.
3. Give the closure of S_1 using set builder notation and labeling with the correct notation.
4. Give the boundary of S_1 using set builder notation and labeling with the correct notation.
5. Is S_1 open, closed, or neither?
6. Determine which of the points from Exercise 1 are accumulation points.
7. Determine which of the points from Exercise 1 are isolated points.



Exercises 1 - 7

8. Consider the set $S_2 = \{e^{\frac{1}{n}i} : n \in \mathbb{Z}\}$.

   (a) Sketch, to the best of your ability, the set S_2.
   (b) Does S_2 have any accumulation points? If so, describe them.
   (c) If there are any accumulation points, are any of them in the set? If so, which ones?
   (d) Are any of the points in the set isolated points? If so, which ones?
   (e) Describe the closure of the set.

9. Let $S_3 = \{e^{\theta i} : \theta \in \mathbb{Q}\}$, where $\mathbb{Q}$ is the rational numbers. Note that *between any two real numbers there is a rational number.*

   (a) Sketch, to the best of your ability, the set S_2.
   (b) Does S_2 have any accumulation points? If so, describe them.
   (c) If there are any accumulation points, are any of them in the set? If so, which ones?
   (d) Are any of the points in the set isolated points? If so, which ones?
   (e) Describe the closure of the set.

10. Do Exercise 2 on page 32 of Churchill and Brown.
11. Do Exercise 3 on page 32 of Churchill and Brown.
12. In this exercise you will compute the difference quotient $\dfrac{f(z) - f(z_0)}{z - z_0}$ for $z_0 = 1 + 3i$ and $f(z) = z^2$, using each value of z given. All answers should be given in $a + bi$ form. Do the first two by hand (*showing all steps*), use your calculator or some software to help you do the other two. **Give the first one in exact form, all others as decimals, rounded to four places past the decimal when rounding is necessary.**

    (a) $z = 2 + 2i$ (b) $z = 1 + 3.1i$ (c) $z = 0.99 + 3.01i$ (d) $z = 1.02 + 2.99i$

13. Use long division to divide $(x^3 - 5x^2 - 13x + 2) \div (x + 2)$. Show your work, of course!