Math 411 ASSIGNMENT 6 **Due at 3 PM Wednesday, January 22nd**

For part of this assignment you will be considering the following sets:

$$S_1 = \{z \in \mathbb{C} : 1 < |z| < 2\} \qquad S_2 = \{z \in \mathbb{C} : 0 < |z| < 2\} \qquad S_3 = \{z \in \mathbb{C} : |z| < 2\}$$

Here is some terminology that is commonly used for sets like these:

- A set of the form $\{z \in \mathbb{C} : |z| < R\}$ for some $R > 0$ is called an **open disk**.
- A set of the form $\{z \in \mathbb{C} : r < |z| < R\}$, where r and R are non-negative real numbers, is called an **annular region**.
- When $r = 0$, such a region is also called a **punctured disk**.

1. Let's use the notation $\text{int}S$ for the interior of a set S. For which of the sets above is $\text{int}\overline{S} = \text{int}S$?
2. For which of the sets is $\overline{\text{int}S} = \overline{S}$?
3. The sets S_1, S_2 and S_3 are all open sets, and they are all connected. For this exercise you might wish to consider sets that are not open, not connected, or both. Give a set S for which $\overline{\text{int}S} = \overline{S}$.
4. I'm not sure whether there are any sets for which $\text{int}\overline{S} = \text{int}S$, but I haven't thought about it too hard or researched it. See if you can find such a set.

Here is some terminology you will now need:

- A continuous path from one point to another in the complex plane is called a **contour**.
- A contour that starts and ends in the same place is called a **closed contour**.
- A closed contour that only intersects itself at the starting and ending points is called a **simple closed contour**.
- A connected open set S is **simply connected** if every simple closed contour within S encloses only points of S. (A connected open set that is not simply connected is called **multiply connected**.)

5. Which of the sets S_1, S_2 and S_3 above are simply connected?
6. For which of the sets S_1, S_2 and S_3 is $\text{int}\overline{S}$ simply connected?
7. One might be inclined to think that an accumulation point of a set is also a boundary point of the set.
   (a) Give, with *BOTH* a picture and set builder notation, a set and a point z where the point is a boundary point but not an accumulation point.
   (b) Give, with *BOTH* a picture and set builder notation, a set and a point z where the point is an accumulation point but not a boundary point. *This exercise might be harder than (a) because the answer is simpler!*
8. We will soon have need to generated specific contours. This is done by mapping a closed interval of the real line into the complex plane via a function. Several such functions are shown below. Sketch the contour described by each, indicating clearly the starting point and ending point.
   (a) $z(t) = (2 + t) + (1 + 3t)i, \quad 0 \le t \le 1$
   (b) $z(t) = (-4 - 2t) + (3 - 7t)i, \quad 0 \le t \le 1$
   (c) $z(\theta) = 3e^{i\theta}, \quad 0 \le \theta \le \frac{\pi}{2}$
   (d) $z(\theta) = (3 + i) + 2e^{i\theta}, \quad 0 \le \theta \le 2\pi$
9. We call something like $z(t) = (2 + t) + (1 + 3t)i, \ 0 \le t \le 1$ a **parametrization** of a contour. Observe your results from Exercises 8(a) and (b) carefully, then give a parametrization for a line segment from $z = 3 - i$ to $z = -2 + 5i$.
10. Now, using your answers to 8(c) and (d) as a guide, give a parametrization for an upper half circle with center $z = 4 + 3i$ and radius five units, traveled counterclockwise.
11. (a) Using long division, divide 1 by $1 - x$, continuing until you have six terms.
    (b) Evaluate both $\dfrac{1}{1-x}$ and the first four terms of your answer to (a) for $x = -2,\ 0,\ \frac{1}{2},\ 1$. **Organize your work and results in some way that is easy for "the reader" (me!) to understand.** For which values of x does it appear that both things would give the same result if you used the "entire answer" to (a)?